A Thesis for the Degree of Master

Iterative Receiver Design
for Coded MIMO Systems
on SC/OFDM Transmission

Wooram Shin
School of Engineering
Information and Communications University
2008
Iterative Receiver Design for Coded MIMO Systems on SC/OFDM Transmission
Iterative Receiver Design
for Coded MIMO Systems
on SC/OFDM Transmission

Advisor : Professor Joonhyuk Kang

by

Wooram Shin

School of Engineering
Information and Communications University

A thesis submitted to the faculty of Information and Communications University in partial fulfillment of the requirements for the degree of Master of Science in the School of Engineering

Daejeon, Korea
December 20. 2007
Approved by

Professor Joonhyuk Kang
Major Advisor
Iterative Receiver Design for Coded MIMO Systems on SC/OFDM Transmission

Wooram Shin

We certify that this work has passed the scholastic standards required by the Information and Communications University as a thesis for the degree of Master

December 20, 2007

Approved:

Chairman of the Committee
Joonhyuk Kang, Assistant Professor
School of Engineering

Committee Member
Joongsoo Ma, Professor
School of Engineering

Committee Member
Wan Choi, Assistant Professor
School of Engineering
Abstract

Recently, researches on multiple-input multiple-output (MIMO) technique deploying multiple transmit and receive antennas have abruptly emerged due to its potential to greatly improve spectral efficiency. Representatively, V-BLAST (Vertical Bell Laboratories Layered Space-Time) architecture has been widely accepted for the fourth generation mobile communications, such as IEEE 802.16e and 3GPP-LTE since it is able to exploit multiplexing gain and diversity gain proportional to the number of transmit antenna and the number of receive antenna, respectively, with ease. The MIMO systems concatenated with channel coding present considerably better error performance. In particular, space-time bit-interleaved coded modulation (ST-BICM) carries flexible rate-operability and rate-compatibility by designing in consideration of code-rate, modulation-level, and the number of transmit antenna. The receiver structure, on the other hand, processing inter-transmit-antenna interference (ITAI) elimination is relatively even more complicated than the transmitter one. Iterative detection and decoding (IDD) technique has been regarded as the most favorable candidate of high performance receiver approaching matched filter bound, which is classified into sphere
detection (SD) and linear minimum mean squared error (LMMSE) detection. The SD scheme based on the tree-searching algorithm achieves optimal performance with requiring high computational burden, while the LMMSE approach performing simple linear operation with a priori information fed back from channel decoder converges the matched filter bound (MFB) after several iterations.

In this thesis, we consider two problems on the iterative receiver design for the coded MIMO systems. The first one is the severe performance degradation due to the imperfect channel state information (CSI) on the single-carrier (SC) transmission over block fading channels. Especially, it is emphasized that the interesting operation signal-to-noise ratio (SNR) range is very low in the ST-BICM systems. In order to resolve this problem, we propose iterative channel estimation, detection, and decoding (ICEDD) technique in a LMMSE fashion with a priori information fed back from the soft-input soft-output (SISO) decoder. In addition, the Bayesian mean-squared-error (MSE) of the channel estimation is taken into account in detection and demapping, improving error performance in higher-order modulation. The computationally efficient LMMSE detection algorithm avoiding matrix inverse operation is proposed using Sherman-Morrison formula as well. Another problem is the performance loss on orthogonal frequency division multiplexing (OFDM) transmission over severely time-varying multipath channels due to the ICI and energy falloff of the desired subcarriers. We propose the IDD in a blockwise fashion, which considers dominantly effective ICIs adjacently around the desired subcarriers, to accomplish error performance close to MFB with reasonable computational complexity. The LMMSE filter matrix, particularly, can be obtained with lower computational burden using the two proposed algorithms: double-pipelining and fast recursion.
Contents

Abstract i

Contents iii

List of Figures vi

List of Tables viii

List of Abbreviations ix

List of Notations xii

1 Introduction 1

1.1 Previous Work and Theoretical Perspective . . . . . . . . 2

1.1.1 Turbo Principle . . . . . . . . . . . . . . . . . . . . 2

1.1.2 Coded MIMO Systems on SC transmission . . . . . . 4

1.1.3 Coded MIMO Systems on OFDM transmission . . . . . 5

1.2 Motivation and Problem Definition . . . . . . . . . . . . . 6

1.3 Thesis Organization . . . . . . . . . . . . . . . . . . . . . 8

2 On the Single-Carrier Systems:

Coded MIMO systems with imperfect CSI over block fading channels 9

2.1 System Description . . . . . . . . . . . . . . . . . . . . . 9

2.2 Proposed ICEDD Receiver . . . . . . . . . . . . . . . . . 12

2.2.1 LMMSE Channel Estimation . . . . . . . . . . . . . 12

2.2.2 LMMSE Detection . . . . . . . . . . . . . . . . . . . 14

2.2.3 Linearized Matrix Inversion Algorithm . . . . . . . 17
3 On the Multi-Carrier Systems:
Coded MIMO-OFDM systems over doubly-selective fading channels
  3.1 System Description ........................................... 29
  3.2 Proposed IDD Receiver ......................................... 33
    3.2.1 Blockwise LMMSE Detection .............................. 33
    3.2.2 Double-Pipelining Algorithm ......................... 35
    3.2.3 Fast Recursive Algorithm .............................. 37
    3.2.4 Soft-Demapping ....................................... 38
  3.3 Numerical Results and Discussion .......................... 39
    3.3.1 Error Performance .................................. 40
    3.3.2 EXIT Chart Analysis .................................. 42
    3.3.3 Complexity Analysis .................................. 43

4 Conclusion and Future Work ........................................ 47

Appendix

A Matrix Inversion ..................................................... 49
  1.1 Matrix Inversion with its Submatrices .................... 49
  1.2 Submatrix Inversion from its Inverse Matrix ............. 50

B Statistics of ICI ...................................................... 51
  2.1 Energy Term of ICI .......................................... 51
  2.2 Colored Term of ICI ......................................... 52
List of Figures

1.1 General turbo decoder block diagram. . . . . . . . . . . . . . . 4

2.1 Transceiver block diagram for coded MIMO systems with
ST-BICM on SC transmission. . . . . . . . . . . . . . . . . . . . 11

2.2 BER Performance comparison of IDD vs. MFB for QPSK
with perfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $L = 72$, and
$\eta = 4$ bps/Hz). . . . . . . . . . . . . . . . . . . . . . . . . 20

2.3 BER Performance comparison of IDD vs. MFB for 16-
QAM with perfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $L =
72$, and $\eta = 8$ bps/Hz). . . . . . . . . . . . . . . . . . . . 21

2.4 BER Performance comparison of ICEDD vs. IDD for
QPSK with imperfect CSI ($N_R = N_T = 4$, $R_C = 1/2$,
$L = 72$, and $\eta = 4$ bps/Hz). . . . . . . . . . . . . . . . . . . . 22

2.5 BER Performance comparison of ICEDD vs. IDD for
QPSK with imperfect CSI ($N_R = N_T = 4$, $R_C = 1/2$,
$L = 72$, and $\eta = 8$ bps/Hz). . . . . . . . . . . . . . . . . . . . 23

2.6 EXIT charts of MIMO demapper and MAP channel de-
coder for QPSK ($N_R = N_T = 4$, $R_C = 1/2$, $L = 72$, and
$\eta = 4$ bps/Hz). . . . . . . . . . . . . . . . . . . . . . . . . . 24

2.7 EXIT charts of MIMO demapper and MAP channel de-
coder for 16-QAM ($N_R = N_T = 4$, $R_C = 1/2$, $L = 72$,
and $\eta = 8$ bps/Hz). . . . . . . . . . . . . . . . . . . . . . . . . 25

3.1 Example of CFR energy distribution. . . . . . . . . . . . . . 29

3.2 Transceiver block diagram for coded MIMO-OFDM sys-
tems with ST-BICM . . . . . . . . . . . . . . . . . . . . . . . . 30
3.3 Example of banded effective CFR matrix. .................. 33
3.4 BER Performance comparison of ICEDD vs. IDD for QPSK with imperfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $N = 128$, and $\eta = 4$ bps/Hz). .................. 40
3.5 BER Performance comparison of ICEDD vs. IDD for QPSK with imperfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $N = 128$, and $\eta = 4$ bps/Hz). .................. 41
3.6 EXIT charts of MIMO demapper and MAP channel decoder for QPSK ($N_R = N_T = 4$, $R_C = 1/2$, $N = 128$, and $\eta = 4$ bps/Hz). .................. 43
3.7 EXIT charts of MIMO demapper and MAP channel decoder for QPSK ($N_R = N_T = 4$, $R_C = 1/2$, $N = 128$, and $\eta = 4$ bps/Hz). .................. 44
List of Tables

1.1 Examples of Serially-Concatenated Communication Systems ........................................... 3

2.1 Signal Mapping and Statistics with A Priori Information$^1$ 10

3.1 Matrix Computational Complexity Evaluation Functions$^2$ 45
3.2 Examples of Complexity for the Proposed Algorithms .......................... 46
List of Abbreviations

AWGN  Additive White Gaussian Noise
BCJR  Bahl-Cocke-Jelinek-Raviv
BER  Bit Error Rate
BICM  Bit-Interleaved Coded Modulation
BPSK  Binary Phase Shift Keying
CDMA  Code Division Multiple Access
CFR  Channel Frequency Response
CP  Cyclic Prefix
Dec  Decoder
Enc  Encoder
FEC  Forward Error Correction
FFT  Fast Fourier Transform
ICEDD  Iterative Channel Estimation, Detection, and Decoding
ICI  Inter-Carrier Interference
IDD  Iterative Detection and Decoding
IFFT  Inverse Fast Fourier Transform
i.i.d.  independently and identically distributed
**ISI** Inter-Symbol Interference

**LDPC** Low Density Parity Check

**LLR** Log-Likelihood Ratio

**LMMSE** Linear Minimum Mean Squared Error

**MAP** Maximum A Posteriori Probability

**MFB** Matched Filter Bound

**MIMO** Multiple-Input Multiple-Output

**MMSE** Minimum Mean Squared Error

**MSE** Mean Squared Error

**OFDM** Orthogonal Frequency Division Multiplexing

**P/S** Parallel to Serial Converter

**PSK** Phase Shift Keying

**QAM** Quadrature Amplitude Modulation

**QPSK** Quadrature Phase Shift Keying

**SC** Single-Carrier

**SD** Sphere Detection

**SINR** Signal-to-Interference-Plus-Noise Ratio

**SISO** Soft-Input Soft-Output

**SNR** Signal-to-Noise Ratio

**S/P** Serial to Parallel Converter
ST-BICM  Space-Time Bit-Interleaved Coded Modulation
ST-TCM  Space-Time Trellis Coded Modulation
V-BLAST  Vertical Bell Laboratories Layered Space-Time
WSSUS  Wide-Sense Stationary Uncorrelated Scattering
List of Notations

\( a \) scalar  
\( \mathbf{a} \) vector  
\( \mathbf{A} \) matrix  
\( \mathbf{A}_{[M \times N]} \) the matrix of size \( M \times N \)  
\( \mathbf{A}_{m_1:m_2,n_1:n_2} \) the submatrix of \( \mathbf{A} \) from \( m_1 \)-th row and \( n_1 \)-th column to \( m_2 \)-th row and \( n_2 \)-th column  
\( a_{ij} \) the \( i \)-th row and \( j \)-th column element of \( \mathbf{A} \)  
\( \mathbf{a}_i \) the \( i \)-th column of \( \mathbf{A} \)  
\( \mathbf{I}_\alpha \) the \( \alpha \times \alpha \) identity matrix  
\( \mathbf{0}_{\alpha,\beta} \) the \( \alpha \times \beta \) all zero matrix  
\( \cdot^* \) complex conjugate  
\( \cdot^T \) transpose  
\( \cdot^H \) Hermitian transpose  
\( \cdot^{-1} \) inverse  
\( \text{diag}(\cdot) \) diagonal matrix  
\( \mathbf{R}_x \triangleq \mathbf{E}[\mathbf{x}\mathbf{x}^H] \)  
\( \mathbf{R}_{xy} \triangleq \mathbf{E}[\mathbf{x}\mathbf{y}^H] \)
\( C_x \triangleq \mathbb{E}[x x^H] - \mathbb{E}[x] \mathbb{E}[x^H] \) the auto-covariance matrix of \( x \)

\( C_{xy} \triangleq \mathbb{E}[x y^H] - \mathbb{E}[x] \mathbb{E}[y^H] \) the cross-covariance matrix of \( x \) and \( y \)

\( \psi_{\mu,\sigma^2}(y) \triangleq \frac{1}{\pi \sigma^2} e^{-|y-\mu|^2/\sigma^2} \) the complex Gaussian distribution with mean \( \mu \) and variance of \( \sigma^2 \)

\( J_0(\cdot) \) the zeroth-order Bessel function of the first kind

\( \delta(n) \triangleq \begin{cases} 
1 & \text{if } n = 0, \\
0 & \text{if } n \neq 0
\end{cases} \) the Kronecker delta function

\( \iota \triangleq \sqrt{-1} \) the imaginary unit
1. Introduction

Recent mobile digital communication systems toward the fourth generation are desired to fulfill the high data rate transmission providing smoothly multimedia services with high reliability. One promising way to accomplish the high spectral efficiency without more power consumption and bandwidth sacrifice is multiple-input multiple-output (MIMO) technique [1]. Space-time codes deploying multiple antennas at both transmitter and receiver are capable of exploiting multiplexing gain and diversity gain with a trade-off [2, 3]. There have been many studies on space-time architecture jointly combined with channel codes due to its robustness to fading environment. In addition, MIMO systems for broadband mobile communications have been combined with orthogonal frequency division multiplexing (OFDM) technique since it is considered as the most promising technology to offer high spectral efficiency in conjunction with robustness to frequency selective fading channels [4–6]. Accordingly, coded MIMO-OFDM systems achieving frequency diversity as well as spatial diversity are more sophisticated for the next-generation wireless communication systems than the single-carrier system. In this thesis, we consider coded MIMO systems on SC- and OFDM-transmission. The coded MIMO systems is extensively classified into two categories depending upon a manner of encoding and mapping: ST-TCM and ST-BICM. The former jointly performs channel coding, modulation, and space-time coding as a function block while they are separately conducted by employing a bit-interleaver between channel encoder and signal mapper in the latter [7–10]. The ST-BICM architecture is more practically efficient due to its simplicity and flexible rate-compatibility in comparison to the ST-TCM, so the ST-TCM has
not been adopted in most of the standardization for the next generation mobile communication systems such as IEEE 802.16e and 3GPP-LTE. In this respect, we focus on the ST-BICM with iterative receiver design aiming at yielding optimum error performance in a computationally efficient manner.

1.1 Previous Work and Theoretical Perspective

In this section, the overview of turbo principle is presented skimming over its history, theory, and applications. In the following, we study the previous researches on the coded MIMO systems on SC- and OFDM-transmission with their theoretical perspective in terms of diversity, coding, and multiplexing gain. In the following,

1.1.1 Turbo Principle

Prior to investigate the previous work related to the coded MIMO systems, we briefly overview the turbo principle in a historical order. Basically, iterative receiver has been motivated by the turbo principle inspired from turbo engine since the output values of one decoder is fed back to the input values of the other decoder. In 1949, Shannon proved channel capacity theorem which is a maximum bound of communication link under the error-free hypothesis with entropy concept [11]. Then Gallager, in 1962, discovered LDPC with iterative decoding, which has been recently rediscovered with graph-theoretical approaches [12]. In 1966, Forney introduced concatenated codes approaching closely Shannon’s limit with computationally inefficient decoding process. In 1993, Berrou et al. proposed turbo codes requiring practically low computational complexity with performance close to Shannon’s limit by 0.5 dB, which has abruptly provoked researches on iterative decoding [14]. Douillard et al. applied the iterative decoding concept into equalization
Table 1.1: Examples of Serially-Concatenated Communication Systems

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Outer Coder</th>
<th>Inner Coder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concatenated</td>
<td>FEC Enc/Dec</td>
<td>FEC Enc/Dec</td>
</tr>
<tr>
<td>LDPC</td>
<td>Check Nodes</td>
<td>Variable Nodes</td>
</tr>
<tr>
<td>BICM</td>
<td>FEC Enc/Dec</td>
<td>Mapper and Demapper</td>
</tr>
<tr>
<td>Multiuser</td>
<td>FEC Enc/Dec</td>
<td>FEC Enc/Dec</td>
</tr>
<tr>
<td>Equalization</td>
<td>FEC Enc/Dec</td>
<td>ISI Channel and Detector</td>
</tr>
<tr>
<td>MIMO</td>
<td>FEC Enc/Dec</td>
<td>Mapper and Detector</td>
</tr>
<tr>
<td>Source-Channel</td>
<td>Source Enc/Dec</td>
<td>FEC Enc/Dec</td>
</tr>
</tbody>
</table>

on ISI channels in 1995 [15]. The turbo principle was initially regarded as general receiver technique for the communication systems by Hagenauer in 1997 [16]. The turbo principle is utilized in the serially- or parallely-concatenated structure between two communication function blocks, which are linked by the interleaver and the deinterleaver. The parallely-concatenated architecture is not practically used since the different kinds of communication function block are serially consecutive in usual. On the other hand, the practicability of the serially-concatenated systems is quite flexible in a various fashion as summarized in Table 1.1. We consider the general serially-concatenated structure depicted in Figure 1.1. Herein, $L$-value denotes the LLR of binary random variable defined as follows

$$L(c) \triangleq \ln \frac{P(c = +1)}{P(c = -1)}.$$  \hspace{1cm} (1.1)

Its sign means the hard decision value, and the absolute value signifies the reliability on the decision. In general communication systems, the message is transmitted over the distortion of channel or noise. This motivates introducing MAP algorithm which literally maximizes a posteriori probability. In other words, we measure the statistic of the
original message conditioned on the received message as

\[ L_D(c) \triangleq \ln \frac{P(c = +1|r)}{P(c = -1|r)} = \ln \frac{P(r|c = +1)}{P(r|c = -1)} + \ln \frac{P(c = +1)}{P(c = -1)} \triangleq L_E(c) \triangleq L_A(c) \]  

(1.2)

where \( L_D(c) \), \( L_A(c) \), and \( L_E(c) \) denote a posteriori LLR, a priori LLR, and extrinsic LLR, respectively. The a priori LLR, initially, is equal to zero since it is considered as a unknown value without any statistical information at the receiver side. The turbo principle, fortunately, enables to refine this probabilistic information in a iterative fashion as depicted in Figure 1.1. The extrinsic LLR from the inner decoder turns into the a priori LLR of the outer decoder through deinterleaver (or interleaver in the opposite case), vice and virtue. After several iterations, the reliability of this value is enhanced enough to achieve correct decision.

1.1.2 Coded MIMO Systems on SC transmission

The coded MIMO systems with ST-BICM on SC transmission was proposed by Tonello in 2000 [7], which is considered as the most prospec-
ative technique since its structure concatenating channel encoder, interleaver, and signal mapper is quite simple with considerably reliable performance and rate compatibility is very flexible by controlling the code rate and the number of transmit antennas [7–10]. The theoretical aspects of this architecture were analyzed in terms of diversity and coding gain [7]. Over block fading channels, the diversity gain is given by \(N_T \cdot N_R\), and the coding gain is maximized by the product distance of the Hamming distance and the minimum squared Euclidean interdistance. Over fast fading channels, on the other hand, the diversity gain is simplified as the multiplication of \(N_R\) and the Hamming distance. The optimum coding gain is evaluated by the joint maximization of the Hamming distance and the minimum squared Euclidean interdistance. The transmitter can be designed with consideration of those factors to maximize appropriately diversity and coding gain. Meanwhile, a relatively complicated iterative receiver structure exchanging \textit{a priori} information between the MIMO demapper and the channel decoder is required to obtain optimal performance. The iterative receivers based on turbo principle has been applied to multifarious communication systems like inter-symbol-interference (ISI) equalization [17] and multiuser code division multiple access (CDMA) [18] as mentioned in the previous subsection. Iterative technique for ST-BICM systems is mainly classified into two types: sphere detection [19, 20] and linear minimum mean squared error (LMMSE) detection [8, 21, 22]. While the sphere detection based iterative receiver yields optimal performance with high computational complexity, the LMMSE iterative receiver converges to it in a few iterations with reasonably low complexity.

### 1.1.3 Coded MIMO Systems on OFDM transmission

After ST-BICM technique on SC transmission was introduced, it has been applied to the multi-carrier systems, OFDM [9,10]. The detection
and decoding strategy is similar to the SC transmission. Its theoretical respect, however, is distinctively different from that of SC transmission as follows [10]. We assume that the $K$ multipath channel taps have equal power, then the diversity order $D$ is given by

$$D = \min(L, KN_T) \cdot N_R$$  \hspace{1cm} (1.3)

where $L$ denotes the effective code length related to the Hamming distance. Eq. (1.3) gives rise to some implications. Over flat fading channels (identically, $K = 1$), the diversity order becomes simply $N_T \cdot N_R$, which means that the number of transmit antenna is a momentous factor directly affecting error performance. For frequency-selective fading channels with large delay spread (especially, $K \cdot N_T > L$), on the contrary, the diversity order depends only on $L \cdot N_T$. This implicates that employing more transmit antennas is no more significant due to its saturation appearance. Therefore, other gain, called coding gain, can be regarded as more important requisite to determine transmit parameters. The coding gain can be optimized by maximizing free Hamming distance of the channel coding as a basic rule of thumb. Finally, the multiplexing gain is proportional to $N_T$ by delivering different data from each transmit antenna.

1.2 Motivation and Problem Definition

The previous researches on the iterative detection and decoding (IDD) receiver for ST-BICM systems have been worked with assumption that CSI is perfectly known to the receiver side. In realistic environment, meanwhile, imperfect CSI occurs due to the channel estimation errors, which causes the severe performance degradation even though data symbols are transmitted within reasonable SNR range. This can be overcame by data-aided or decision-directed channel estimation [24]-[27]; especially, its necessity is inevitably urgent for ST-BICM systems.
due to the very low interesting operation SNR range. In this respect, channel estimation is iteratively refined in a decision-directed fashion with \textit{a priori} information, then the error performance is considerably improved. In addition, a consideration of Bayesian MSE gives rise to SNR gain, and resolves slight error-floor problem in higher-order modulation. The other contribution is a low-complexity LMMSE detector and demapper without matrix inversion using Sherman-Morrison formula.

On the OFDM transmission over time-varying multipath fading channels, so-called doubly-selective fading channels, however, the orthogonality between subcarriers would be destructed by Doppler effect, which causes the performance degradation due to ICI and energy loss of the desired subcarriers. In this thesis, consequently, IDD technique is applied in order to conquer both ICI and ITAI in a computationally efficient manner. The main problem to resolve them is that the size of effective CFR matrix is proportional to FFT size, which requires exponentially increase of computational complexity to obtain LMMSE filter matrix. Meanwhile, the energy of ICI is dominantly large around adjacent subcarriers, and decreases as far from them. Considering this property, the only several dominant ICIs near the desired subcarrier are piled up for each subcarrier to reduce the size of effective CFR matrix. The another aspect to reduce computational load is that the LMMSE filter is calculated recursively using consecutive pattern of the effective CFR matrix, which makes the IDD technique more practical to be used in real systems. Then the error performance of the proposed algorithm approaches to the MFB in several iterations with significantly low computational complexity.
1.3 Thesis Organization

In this thesis, iterative receiver design for coded MIMO systems on SC- and OFDM-transmission is presented to accomplish optimal error performance in a computational efficient manner. The remaining of this thesis organized as follows. In Chapter 2, we focus on the SC transmission, specifically, the ICEDD is proposed for the coded MIMO systems with imperfect CSI over block fading channels in a practical point of view. Chapter 3 considers the IDD for the coded MIMO-OFDM systems over severely time-varying multipath fading channels degrading error performance due to time-selectivity in spite of resolving frequency-selectivity. The following chapter, finally, wraps up our achievement with suggestions of future work.
2. On the Single-Carrier Systems: Coded MIMO systems with imperfect CSI over block fading channels

In this chapter, we propose the ICEDD with *a priori* information in a LMMSE fashion for the coded MIMO systems on the SC transmission over block fading channels. The LMMSE channel estimation is conducted with pilot symbols at the initial step, and then it exploits soft data symbols extracted from *a priori* LLR for the remaining iterations. In the sequel, the LMMSE detection is performed considering the statistic of the channel estimation errors as well as that of data symbols using probabilistic information. Finally, the soft-demapper evaluating soft information fed into the SISO decoder is derived with Gaussian approximation of the detected symbols. We utilize the soft values for channel estimator and MIMO demapper (signifying MIMO detector and soft-demapper) to minimize uncertainty of temporarily estimated symbols, which reduces the error propagation caused by quantization. The simulation results depict that the performance of the proposed algorithm nearly approaches to that of the genie-aided bound.

2.1 System Description

In this section, we consider a discrete time MIMO channel representation suffered by i.i.d. block fading, which is a valid assumption since the sampling rate is even more rapid than the channel variation rate [20]. The CSI at the receiver side can be estimated with the help of pilot sequences per coding block, and it would be updated by the
Table 2.1: Signal Mapping and Statistics with A Priori Information

<table>
<thead>
<tr>
<th></th>
<th>QPSK</th>
<th>16-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_i$</td>
<td>$\sqrt{2} ((x_{i,1} - 0.5) + i(x_{i,2} - 0.5))$</td>
<td>$\sqrt{8} ((x_{i,1} - 0.5)(-x_{i,2} + 0.5) + i(x_{i,3} - 0.5)(-x_{i,4} + 0.5))$</td>
</tr>
<tr>
<td>$E[s_n]$</td>
<td>$\frac{1}{\sqrt{2}} (\theta_{n,1} + i\theta_{n,2})$</td>
<td>$\frac{2}{\sqrt{10}} (\theta_{n,1} (2 - \theta_{n,2}) + i\theta_{n,3} (2 - \theta_{n,4})$</td>
</tr>
<tr>
<td>$R_{s_n}$</td>
<td>1</td>
<td>$\frac{1}{4} (9 \omega_{n,4} + \omega_{n,2} \omega_{n,4} + \omega_{n,2} \omega_{n,4} + \omega_{n,2} \omega_{n,4})$</td>
</tr>
</tbody>
</table>

$\theta_{n,j} \triangleq \text{tanh} \left( \frac{1}{2} L(c_{n,j}) \right)$, $\omega_{n,j} \triangleq 1 + \theta_{n,j}$, and $\bar{\omega}_{n,j} \triangleq 1 - \theta_{n,j}$.

LMMSE channel estimation with the probabilistic information fed back from the channel decoder to be more accurate. After the transmitted signal undergoes the MIMO channel during $l$-th symbol period, the received signal can be observed as follows

$$r_l = Hs_l + z_l, \quad l \in \{1, 2, \ldots, L\}$$

$$= \hat{H}s_l + H_{\varepsilon}s_l + z_l \quad (2.1)$$

where $H \in \mathbb{C}^{N_r \times N_t}$ is the really propagated channel matrix whose entries have the i.i.d. complex Gaussian distribution with zero mean and unit variance during a block period, $\hat{H} \in \mathbb{C}^{N_r \times N_t}$ is the estimated channel matrix, $H_{\varepsilon} \in \mathbb{C}^{N_r \times N_t}$ is the channel estimation error matrix, $s_l \in \Xi^{N_t \times 1}$ is the transmitted signal vector with unit energy, $z_l \in \mathbb{C}^{N_r \times 1}$ is the i.i.d. complex Gaussian noise vector with zero mean and variance $N_0/2$ per dimension, and $r_l \in \mathbb{C}^{N_r \times 1}$ is the received signal vector. $L$ is the block length, and $\Xi \triangleq \{\xi_1, \xi_2, \ldots, \xi_M\}$ is the constellation symbol set for $M$-ary PSK ($M$-PSK) or $M$-ary QAM ($M$-QAM). The signal mapper for QPSK and 16-QAM with Gray mapping are presented by
the generating function in Table 2.1. The estimated channel matrix $\hat{H}$ refined by the LMMSE channel estimator is augmented as follows

$$\hat{H} \triangleq \begin{bmatrix} \hat{h}_1 & \hat{h}_2 & \cdots & \hat{h}_{N_R} \end{bmatrix}^T$$

(2.2)

where $\hat{h}_n$ is the estimated channel vector corresponding to the $n$-th receive antenna. For the notational convenience, the symbol period index $l$ would be omitted except for the LMMSE channel estimation in Subsection 2.2.1. Figure 2.1 depicts the transmit and receive structure for ST-BICM systems. The binary message bit sequence $m$ from the source is encoded by the binary convolutional code of rate $R_C$, which is followed by the random interleaver. Then, the interleaved-coded bit sequence $c \triangleq \{c_1, c_2, \ldots, c_{L \cdot N_T}\}$ partitioned into $c_t \triangleq \{c_1, c_2, \ldots, c_{\log_2 M}\}$ is mapped onto the symbol vector sequence $s \triangleq \{s_1, s_2, \ldots, s_L\}$, and finally the symbol vector $s_l$ is transmitted from the $N_T$ transmit antennas for each symbol period. The receive procedure is relatively complex than the transmit one due to the iterative steps. The iterative process inspired by the turbo principle is performed by exchanging the a priori LLR $L_A(c_{n,j}) \triangleq \ln \frac{P(c_{n,j}=1)}{P(c_{n,j}=0)}$ between the SISO decoder and the channel estimator / MIMO demapper. Initially, there is no feedback from the SISO decoder, so the channel estimation depends only on the pilot
symbols. Then the LMMSE detection and its soft-demapping is accomplished with $L_A(c_{n,j}) = 0$, which yields the extrinsic LLR $L_E(c_{n,j})$ specifically defined in Eq. (2.19). After the extrinsic LLR sequence $L_E(c)$ is deinterleaved, it is considered as the soft-input value for the SISO decoder. The SISO decoder performed by the BCJR algorithm [33] reckons the a posteriori LLR sequence $L(\tilde{c})$. This output is interleaved in order to be fed back to the channel estimator / MIMO demapper as the a priori LLR. After initial step, the whole procedure is equivalent to the initial one except that the a priori LLR comprises reliable information. Thus, the channel estimation is carried out both with pilot symbols and data symbols, and the MIMO demapper utilizes a priori information providing the more reliable statistics.

### 2.2 Proposed ICEDD Receiver

In this section, we observe the proposed ICEDD receiver comprised by LMMSE channel estimation, LMMSE detection, linearized matrix inversion algorithm for LMMSE filter, and soft-demapping for the LMMSE detection.

#### 2.2.1 LMMSE Channel Estimation

The performance of the pilot-aided channel estimation is dominantly influenced by the noise since the interesting operation SNR range is very low in the ST-BICM systems. Accordingly, the error of the channel estimation only with the pilot symbols can be considerably large due to the noise. This may cause the performance degradation whether the data part is less or more distorted by the noise; thus, we will use the data symbols in the form of the soft-value as well as the pilot symbols. Also, a LMMSE estimation technique is widely used for the parameter estimation due to its simplicity and optimal performance in the Bayesian
MSE sense. Therefore, the LMMSE theory is applied to the pilot- and data-aided channel estimation with the probabilistic information. We rearrange the input-output relation with respect to the channel vector for the $n$-th receive antenna as unknown parameter:

$$
\bar{r}_n = \bar{S}\bar{h}_n + \bar{z}_n, \quad n \in \{1, \ldots, N_R\}
$$

(2.3)

where $\bar{r}_n \triangleq [ \bar{r}_{n1}^T \ r_{n2} \ \cdots \ r_{NL,n} ]^T$, $\bar{S} \triangleq [ S_P^H \ S_D^H ]^H$, $\bar{h}_n \triangleq [ h_{n1} \ h_{n2} \ \cdots \ h_{nN_T} ]^T$, $\bar{z}_n \triangleq [ \bar{z}_{n1}^T \ z_{n2} \ \cdots \ z_{NL,n} ]^T$, $S_P \triangleq \text{diag}\{\tilde{s}_1, \tilde{s}_2, \ldots, \tilde{s}_{N_T}\}$, and $S_D \triangleq \begin{bmatrix}
    s_{1,1} & s_{1,2} & \cdots & s_{1,N_T} \\
    s_{2,1} & s_{2,2} & \cdots & s_{2,N_T} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{L,1} & s_{L,2} & \cdots & s_{L,N_T}
\end{bmatrix}$.

In the above definitions, $\tilde{s}_p$, $\bar{r}_n$, and $\bar{z}_n$ are the pilot symbol for the channel estimate corresponding to the $p$-th transmit antenna, the received signal vector of the pilot symbol transmission from the $n$-th receive antenna, and the noise vector of the pilot symbol transmission from the $n$-th receive antenna, respectively. The pilot symbol transmission is given by

$$
\tilde{R} = \bar{H}S_P + \tilde{Z}
$$

(2.4)

where $\tilde{R} \triangleq [ \tilde{r}_1 \ \tilde{r}_2 \ \cdots \ \tilde{r}_{N_R} ]^T$, $\tilde{Z} \triangleq [ \tilde{z}_1 \ \tilde{z}_2 \ \cdots \ \tilde{z}_{N_R} ]^T$, and the diagonal entries of $S_P$ are known to both of the transmitter and the receiver. Then the LMMSE channel estimation corresponding to the $n$-th receive antenna can be expressed by

$$
\hat{h}_n \triangleq \mathbb{E}[\bar{h}_n] + C_{\tilde{h}_n}^{-1}(\bar{r}_n - \mathbb{E}[\bar{r}_n]) = \mathbb{E}[S^H] (\mathbb{E}[S]\mathbb{E}[S^H] + Q)^{-1}\bar{r}_n
$$

(2.5)

where $\mathbb{E}[\bar{h}_n] = 0_{N_T,1}$, $C_{\tilde{h}_n} = R_{\bar{h}_n}\mathbb{E}[S^H] = \mathbb{E}[S^H]$, $C_{\bar{r}_n} = \mathbb{E}[S\bar{h}_n\bar{h}_n^H S^H] + R_{\bar{z}_n} = \mathbb{E}[S]\mathbb{E}[S^H] + C_S + R_{\bar{z}_n}$, $C_S = \sum_{p=1}^{N_T} \text{diag}\{0_{1,N_T}, C_{s_1,p}, C_{s_2,p}, \ldots, C_{s_{L,p}}\}$.
Rz_n = N_0 I_{N_T+L}, E[\bar{r}_n] = 0_{(N_T+L),1}, and Q = C_S + Rz_n. The matrix inversion with matrix size of \((N_T + L) \times (N_T + L)\) in Eq. (2.5) requires considerably computational load, so we manipulate this equation by Woodbury’s identity\(^2\) resulting the matrix inversion with matrix size of \(N_T \times N_T\) as

\[
\hat{h}_n = \Phi^{-1} E[S^H]Q^{-1} \bar{r}_n
\]  

(2.6)

where \(\Phi \triangleq E[S^H]Q^{-1}E[S] + I_{N_T}\), \(E[S_P] = S_P\), and \(E[s_{l,p}]'s\) in \(E[S_D]\) and \(C_{sl,p}'s\) are acquired using the \textit{a priori} information. The details of these statistics will be presented in Eqs. (2.10) and (2.11). Finally, the Bayesian MSE of \(\hat{h}_n\) which is used to improve the accuracy of the MIMO demapper in the sequel subsections is yielded as [34]

\[
B_{mse}(\hat{h}_n) \triangleq C_{h_n \hat{h}_n} - C_{h_n \bar{r}_n} C_{\bar{r}_n \bar{r}_n} C_{\bar{r}_n \hat{h}_n}
= I_{N_T} - E[S^H](E[SS^H] + N_0 I_{N_T+L})^{-1} E[S]
= I_{N_T} - E[S^H](E[S]E[S^H] + Q)^{-1} E[S]
= I_{N_T} - (I_{N_T} + E[S^H]Q^{-1}E[S])^{-1} E[S^H]Q^{-1} E[S]
= (E[S^H]Q^{-1} E[S] + I_{N_T})^{-1}
= \Phi^{-1}
\]  

(2.7)

\subsection{LMMSE Detection}

In this subsection, we derive the LMMSE detection using the \textit{a priori} information based on the channel estimation result with the consideration of the channel estimation error statistic \(B \triangleq B_{mse}(\hat{h}_n)\). The detected symbol vector with the LMMSE approach is given by

\[
\hat{s} \triangleq E[s] + C_{sr} C_r^{-1}(r - E[r])
= E[s] + C_s \hat{H}^H (\hat{H}C_s \hat{H}^H + R_{(H,s)} + R_z)(r - \hat{H}E[s])
\]  

(2.8)

where \(C_S \triangleq \text{diag}\{C_{s_1}, C_{s_2}, \ldots, C_{s_{N_T}}\}\), \(R_z = N_0 I_{N_R}\), and

\[
2(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}
\]
In the same as the LMMSE channel estimation, the statistics \( E[s_n] \) and \( R_{sn} \) in Eq. (2.8), so called soft-values, are obtained using the probabilistic information from the SISO decoder as

\[
E[s_n] = \sum_{\xi_i \in \Xi} \xi_i P(s_n = \xi_i) \quad (2.10)
\]

\[
C_{sn} = \sum_{\xi_i \in \Xi} |\xi_i|^2 P(s_n = \xi_i) - |E[s_n]|^2 
\]

\[
\triangleq R_{sn}
\]

where

\[
P(s_n = \xi_i) = \prod_{j=1}^{\log_2 M} \left( \frac{1}{2} + (x_{n,j} - 0.5) \cdot \tanh \left( \frac{1}{2} L_A(c_{n,j}) \right) \right)
\]

and \( x_{i,j} \in \{0, 1\} \) is the \( j \)-th bit of the bit sequence \( x_i \triangleq \{x_{i,1}, \ldots, x_{i,\log_2 M} \} \) corresponding to the constellation symbol \( \xi_i \). To reduce their computational complexity in case of QPSK and 16-QAM, the above statistics
are simply expressed in Table 2.1 due to the symmetry of Gray mapping. Since the detected symbol from the $n$-th transmit antenna would be independent from the a priori information which leads to $E[s_n] = 0$ and $C_{sn} = 1$, we decompose the detected symbol vector in Eq. (2.8) to the detected symbols for each transmit antenna:

$$
\hat{s}_n = E[s_n] + C_{sn} \hat{h}_n^H (\hat{H}C_s\hat{H}^H + R_{(H,s)} + R_z)^{-1} (r - \hat{H}E[s])
$$

$$
= \hat{h}_n^H (\Lambda_n + (1 - C_{sn})\hat{h}_n\hat{h}_n^H)^{-1} (r - \hat{H}E[s] + E[s_n]\hat{h}_n)
$$

(2.12)

where $\Lambda_n \triangleq \hat{H}C_s\hat{H}^H + R_{(H,s)} + (\alpha_n + N_0)I_{NR}$ and

$$
\alpha_n \triangleq (1 - E[s_n s_n^*])b_{mn} - \sum_{p=1, p \neq n}^{N_T} 2 \text{Re}\{E[s_n]E[s_p^*]b_{np}\}. 
$$

The detailed derivations are not provided here due to the lack of space. From Eq. 2.12, an important physical interpretation can be led, which is how much the channel effect corresponding to the $n$-th transmit antenna is reflected depending on the variance of data symbol $C_{sn}$ to minimize the error propagation. The most of computational burden is incurred by the matrix inversion in Eq. (2.12), and there requires these $N_T$ operations for all symbols. However, we can reduce the computational complexity using Woodbury’s identity with the approximation of $\alpha_n$ as follows

$$
\hat{s}_n \approx \hat{h}_n^H (\Lambda + (1 - C_{sn})\hat{h}_n\hat{h}_n^H)^{-1} (r - \hat{H}E[s] + E[s_n]\hat{h}_n)
$$

$$
= \hat{h}_n^H (\Lambda^{-1} - \Lambda^{-1}\hat{h}_n((1 - C_{sn})^{-1} + \hat{h}_n^H\Lambda^{-1}\hat{h}_n)^{-1}
$$

$$
\times \hat{h}_n^H\Lambda^{-1})(r - \hat{H}E[s] + E[s_n]\hat{h}_n)
$$

$$
= W_n \hat{h}_n^H\Lambda^{-1}(r - \hat{H}E[s] + E[s_n]\hat{h}_n)
$$

(2.13)

where $\Lambda \triangleq \hat{H}C_s\hat{H}^H + R_{(H,s)} + (\alpha + N_0)I_{NR}$,

$$
\alpha_n \approx \alpha \triangleq \frac{1}{N_T} \sum_{n=1}^{N_T} \alpha_n, \quad \text{and} \quad W_n \triangleq \frac{1}{1 + (1 - C_{sn})\hat{h}_n^H\Lambda^{-1}\hat{h}_n}. 
$$

Consequently, the number of matrix inverse operation to obtain the detected symbols for all transmit antennas becomes one.
2.2.3 Linearized Matrix Inversion Algorithm

In the previous subsection, the matrix inverse operation is required to perform the LMMSE detection, which occupies harsh computational resources. Over block fading channels, fortunately, a characteristic that the channel matrix is regular during block period would be utilized to avert matrix inversion using Sherman-Morrison formula\(^3\). To apply this property in a computationally efficient manner, Eq. (2.13) can be rewritten as

\[
\hat{s}_n = W_n \hat{h}_n^H \Lambda^{-1} \left( r - \hat{H}E[s] + E[s_n] \hat{h}_n \right)
= W_n \hat{h}_n^H (\hat{H}C_s \hat{H} + \beta I_{N_R})^{-1} \left( r - \hat{H}E[s] + E[s_n] \hat{h}_n \right)
= \frac{W_n}{\beta} \left( \frac{1}{\beta} \hat{H}^H \hat{H} + C_s^{-1} \right)^{-1} \hat{H}^H (r - \hat{H}E[s] + E[s_n] \hat{h}_n)
= W_n \Sigma^{-1} \hat{H}^H (r - \hat{H}E[s] + E[s_n] \hat{h}_n)
\]

(2.14)

where \( \beta I_{N_R} \triangleq R_{(H,s)} + \alpha I_{N_R} + R_z \) and \( \Sigma \triangleq \hat{H}^H \hat{H} + \beta C^{-1}_s \). Then the inverse operation \( \Sigma^{-1} \) is obtained by

\[
\Sigma^{-1} = \left( \hat{H}^H \hat{H} + \sum_{n=1}^{N_T} \frac{\beta}{C_n} \bar{e}_n \bar{e}_n^T \right)^{-1}
= \bar{\Lambda} - \sum_{n=1}^{N_T} \frac{\beta}{C_n} \bar{e}_n \bar{e}_n^T \bar{\Lambda}
= \bar{\Lambda} - \sum_{n=1}^{N_T} \frac{\bar{\Lambda}_n \bar{\Lambda}_n^H}{C_n / \beta + \lambda_{nn}}
\]

(2.15)

where \( \bar{\Lambda}_n \) denotes basis unit vector whose \( n \)-th element is one and the others are zero, and \( \bar{\Lambda} \triangleq (\hat{H}^H \hat{H})^{-1} \). Consequently, Eq. (2.15) is constituted only with the linear combination of \( \bar{\Lambda} \). A comparison of computational complexity between the original matrix inversion and the fast recursive algorithm will be appeared in Subsection 2.3.3.

\(^3\)(A + uv^H)^{-1} = A^{-1} - \frac{A^{-1}uv^HA^{-1}}{1 + v^HA^{-1}u}$
2.2.4 Soft-Demapping

Finally, the extrinsic LLR which would be fed into the SISO decoder as the *a priori* information is derived with the Gaussian approximation given by [18]

\[
p(\hat{s}_n | s_n = \xi_i) \approx \psi_{\mu_{n,i}, \sigma_{n,i}^2}(\hat{s}_n)
\]

where \(\mu_{n,i} \triangleq E[\hat{s}_n | s_n = \xi_i]\) and \(\sigma_{n,i}^2 \triangleq C_{\hat{s}_n | s_n = \xi_i}\). Then these two conditional statistics are evaluated as follows

\[
\mu_{n,i} = W_n \hat{h}_n^H \Lambda^{-1}(E[r | s_n = \xi_i] - \hat{H}E[s] + E[s_n] \hat{h}_n)
\]

(2.17)

\[
\sigma_{n,i}^2 = W_n^2 \hat{h}_n^H \Lambda^{-1} C_{r | s_n = \xi_i} \Lambda^{-1} \hat{h}_n
\]

(2.18)

where

\[
C_{r | s_n = \xi_i} = \Lambda - \alpha I_{N_R} - C_{s_n} \hat{h}_n \hat{h}_n^H + (\xi_i \xi_i^* - E[s_n s_n^*]) b_{mn} I_{N_R}
\]

\[
+ \sum_{p=1}^{N_T} \sum_{p \neq n} 2 \text{Re}\{\xi_i E[s_p] - E[s_p] E[s_p^*] b_{np}\} I_{N_R}
\]

\[
= \Lambda - \alpha I_{N_R} + \alpha_n I_{N_R} - C_{s_n} \hat{h}_n \hat{h}_n^H + (\xi_i \xi_i^* - 1) b_{nn} I_{N_R}
\]

\[
+ \sum_{p=1}^{N_T} \sum_{p \neq n} 2 \text{Re}\{\xi_i E[s_p^*] b_{np}\} I_{N_R}.
\]

Based on Eqs. (2.17) and (2.18), the extrinsic LLR \(L_E(c_{n,j})\) can be calculated with the max-log approximation [33] as

\[
L_E(c_{n,j}) \triangleq \ln \frac{P(c_{n,j} = 1 | \hat{s}_n)}{P(c_{n,j} = 0 | \hat{s}_n)} - L_A(c_{n,j})
\]

\[
= \ln \frac{\sum_{\forall \xi_i : x_{i,j} = 1} P(\hat{s}_n | s_n = \xi_i) \prod_{\forall k \neq j} P(c_{n,k} = x_{i,k})}{\sum_{\forall \xi_i : x_{i,j} = 0} P(\hat{s}_n | s_n = \xi_i) \prod_{\forall k \neq j} P(c_{n,k} = x_{i,k})}
\]

\[
\approx \max_{\forall \xi_i : x_{i,j} = 1} v_{n,j,i} - \max_{\forall \xi_i : x_{i,j} = 0} v_{n,j,i}
\]

(2.19)

where \(v_{n,j,i} \triangleq -\frac{|\hat{s}_n - \mu_{n,i}|^2}{\sigma_{n,i}^2} + \sum_{\forall k : k \neq j} (x_{i,k} - 0.5) \cdot L_A(c_{n,k})\).
2.3 Numerical Results and Discussion

In this section, the performance of the ICEDD over block fading channels is demonstrated by the Monte-Carlo simulations. We will compare the performance of the proposed algorithm and the iterative detection and decoding without channel estimation step (IDD) with two ways: average BER curves and EXIT charts. Specifically, there are three kinds of the ICEDD schemes: ICEDD without the channel estimation error statistic (ICEDD1), ICEDD with the channel estimation error statistic with the approximation of $\alpha_n$ (ICEDD2), and ICEDD with the channel estimation error statistic (ICEDD3). Four transmit and receive antennas $N_T = N_R = 4$ are deployed without spatial correlation, and a non-recursive convolutional code of rate $R_C = 1/2$ with polynomials $(133, 171)$ in octal form is employed. Then the spectral efficiency $\eta \triangleq N_T \cdot R_C \cdot \log_2 M$ for QPSK and 16-QAM is 4 bps/Hz and 8 bps/Hz, respectively. 282 and 570 information bits are assigned per coding block for QPSK and 16-QAM, respectively, and an interleaving is performed with 10 coding blocks. Accordingly, the block length $L$ both for QPSK and 16-QAM is equal to 72, and the interleaving size becomes 5640 and 11520, respectively. $N_T$ orthogonal pilot symbols per coding block are used for the initial channel estimation with boosting gain of 2.5 dB. Herein, we define SNR as $E_b/N_0$.

2.3.1 Error Performance

Figure 2.2 shows the BER performance of IDD for QPSK in perfect CSI (PCSI) situation compared with that of MFB or genie-aided bound. The performance almost converges after three iterations (the number of iteration is specified within parenthesis on the simulation curves); especially, a performance loss in SNR is less than 0.5 dB at BER of $10^{-6}$ after only two iterations. Figure 2.3 depicts the BER performance
Figure 2.2: BER Performance comparison of IDD vs. MFB for QPSK with perfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $L = 72$, and $\eta = 4$ bps/Hz).

of IDD for 16-QAM in the same as the QPSK case. The convergence to the MFB is made after four iterations, and the performance gap from the MFB at the low SNR regime is relatively larger than that for QPSK due to the more sensitivity to fading environment.

Figure 2.4 represents the performance comparison between ICEDDs and IDD for QPSK in imperfect CSI (IPCSI) case. The performance improvement of IDD through every iteration is not distinctly large, whose BER is higher than $10^{-3}$ at SNR of 6 dB. This is because the channel estimation error significantly dominates the performance due to the
very low operation SNR range even if the data symbols are transmitted within reasonable SNR. It is resolved by the decision-directed channel estimation for every iteration step with soft information from the channel decoder. On the other side, the performance of ICEDDs after four iterations nearly approaches to the MFB with an SNR loss of about 0.4 dB at BER of $10^{-6}$. In detail, the performance of ICEDD2 and ICEDD3 is not distinctively better than that of ICEDD1 despite considering channel estimation statistics. This is because that the former term of $\alpha_n$, $(1 - \mathbb{E}[s_n s_n^*])b_{nn}$, in Eq. (2.12) would be neglected due to
Figure 2.4: BER Performance comparison of ICEDD vs. IDD for QPSK with imperfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $L = 72$, and $\eta = 4$ bps/Hz).

$E[s_n s_n^*] = 1$ for QPSK.

Figure 2.5 shows the performance comparison for 16-QAM in the same manner of QPSK case. In difference with QPSK, the IDD yields reasonable performance since the SNR of pilot symbols for 16-QAM is larger than that for QPSK due to its higher operation SNR range. However, the performance of ICEDDs is still better than that of IDD. The SNR gain of ICEDDs compared to IDD at BER of $10^{-6}$ is about 1.2 dB. In difference with QPSK, in addition, the performance of ICEDD2...
Figure 2.5: BER Performance comparison of ICEDD vs. IDD for QPSK with imperfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $L = 72$, and $\eta = 8$ bps/Hz).

and ICEDD3 is considerably better than that of ICEDD1 in high SNR regime without slight error-floor, and there is no performance gap between ICEDD2 and ICEDD3. Finally, the performance loss of ICEDD2 after six iterations compared to the MFB at BER of $10^{-8}$ is about 0.3 dB.
**2.3.2 EXIT Chart Analysis**

In this subsection, we shows the EXIT charts of the MIMO demapper and the SISO channel decoder with MAP algorithm, which is a semi-analysis tool to observe the convergence behavior of iterative decodings and the performance limitation [31, 32] with the extrinsic information transfer functions. We generate the input mutual information with assumption that the \textit{a priori} LLR has Gaussian distribution and its mean value is given by half of the variance [31].

Figure 2.6 depicts the EXIT charts for QPSK at SNR of 8 dB under
three different cases: IDD with perfect CSI, ICEDD2 with imperfect CSI, and IDD with imperfect CSI. The ICEDD2 and IDD for imperfect CSI case starts from the same point. Meanwhile, a slope of the ICEDD2 is more gradual than that of the IDD, and a trajectory of the ICEDD2 converges to the limit of the IDD with perfect CSI. This means that the performance of ICEDD2 can approach to that of the perfect channel estimation case while the performance of IDD is limited no matter how many iteration steps may be performed due to the large channel estimation error without refinement.
In Figure 2.7, the EXIT charts for 16-QAM at SNR of 10 dB are presented under the three cases mentioned in Figure 2.6. The tendency of three different EXIT charts is similar to that for QPSK. The mutual information of ICEDD converges to that of IDD after three iterations while the BER performance of ICEDD2 is far from that of MFB even after four iterations. This discrepancy is caused by the generation of input mutual information which is actually evaluated for BPSK; consequently, this overestimates the a priori LLR for 16-QAM.

2.3.3 Complexity Analysis

In this subsection, we evaluate the computational complexity for the computation of the data symbol statistics, the LMMSE channel estimation, and the LMMSE detection. In (2.10, 2.11), the asymptotic computational complexity of the closed-form of the data symbol statistics for QPSK and 16-QAM in Table 2.1 is equal to $O(1)$ by the symmetry of Gray mapping whereas that of the direct computation is given by $O(M \cdot \log_2 M)$. In order to perform the channel estimator and the MIMO demapper for every iteration, the $LN_T$ data symbol statistics should be calculated, which is harsh for the higher-order modulation. Therefore, the closed-form expression is practically suitable in a real system implementation. The main computational burden of the LMMSE channel estimation in Eq. (2.6) is caused by $\Phi^{-1}$, so its asymptotic computational complexity is obtained by $O(N_R N_T^3)$. Finally, the computational complexity of the LMMSE detection in Eq. (2.12, 2.13) is appraised in the following. The number of the matrix inverse operation in Eq. (2.12) is given by $LN_T$, but, after the approximation of $\alpha_n$ and the mathematical manipulation, that in Eq. (2.13) is reduced to $L$. Consequently, the asymptotic computational complexity is equal to $O(LN_T^2)$. In Subsection 2.2.3, particularly, the linearized matrix inversion algorithm is proposed to avoid the direct matrix inverse operation.
With the regularity of the channel matrix over block fading channels, the matrix inverse operation is equivalently modified as Eq. (2.15). Then the exact computational complexity [35] of Eqs. (2.13) and (2.13) is compared with respect to real number operation (assuming that the computational loads of real addition, real multiplication, and real division are identical) as follows

Eq. (2.13) : \( \frac{20}{3} N^3_T + 8N^2_T N_R + 7N^2_T - \frac{8}{3} N_T \),

Eq. (2.15) : \( (10N^3_T + 2N_T) + \frac{1}{L} \left( \frac{20}{3} N^3_T + 8N_R N^2_T + 7N^2_T - \frac{14}{3} N_T \right) \).

Since \( N_R \) is usually greater than \( N_T \), the proposed algorithm can reduce its computational burden approximately as 32% for \( N_R = N_T \) (e.g., the 36% complexity reduction for \( N_R = N_T = 4 \) and \( L = 72 \), and the 57% complexity reduction for \( N_R = 4 \), \( N_T = 2 \), and \( L = 72 \)).
3. On the Multi-Carrier Systems: Coded MIMO-OFDM systems over doubly-selective fading channels

On the OFDM transmission, the time-varying effect gives rise to ICIs degrading error performance due to orthogonality destruction, particularly, the influence of ICIs is more poignant when deploying multiple antennas since they are generated in proportion to the number of transmit antenna. Deliberating on this problem, we should consider the ICI term to overcome performance loss, whose representative optimal solution is the MAP algorithm. However, the MAP algorithm considering all $NN_T \cdot \log_2 M$ coded bits can not be applicable due to its tremendous computational complexity exponentially increasing with the coded bit length. The energy of ICI, fortunately, becomes decreasing as far from the subcarrier corresponding to the diagonal of the effective CFR matrix, depicted Figure 3.1 [28–30]. Using this characteristic, we take dominantly effective $q$ ICIs before and after each subcarrier, respectively, in a banded matrix form. The impact of the remaining ICIs can be combatted by the confidence of error correction capability with channel coding. In the following, the proposed detection algorithms will be given with the banded CFR matrix. The simulation results demonstrate that the proposed algorithm shows error performance close to MFB after several iterations with EXIT chart analysis.
Figure 3.1: Example of CFR energy distribution.

3.1 System Description

Coded MIMO-OFDM systems with code-rate of $R_C$, $N_T$ transmit antennas, $N_R$ receive antennas, and FFT size of $N$ are considered, based on discrete-time baseband representation. Figure 3.2 describes the transceiver block diagram for the coded MIMO-OFDM systems with ST-BICM similar to the single-carrier systems except that OFDM modulation and demodulation are supplementarily performed at the before of transmit antenna modules and the after of receive antenna ones, respectively. Also, the length of the coded bit sequence becomes $N \cdot N_T$ enabling to transmit a coding block during an OFDM symbol period. In
this respect, the symbol period index \( l \) and the block length \( L \) in Chapter 2 are replaced by the subcarrier index \( m \) or \( k \) and the FFT size \( N \), respectively. We briefly overview the transmit structure previously mentioned in the SC systems with assumption that the previous notations are retained identically. The binary message bit sequence is encoded by the binary convolutional code, followed by the random interleaver. In the sequel, the interleaved-coded bit sequence is parallelized to the \( N_T \) interleaved-coded bit sequences, and each sequence are mapped onto the frequency-domain data symbols over \( N \) subcarriers by \( M \)-PSK or \( M \)-QAM. Finally, the \( N \) frequency-domain data symbols for each transmit antenna are transformed to the \( N \) time-domain data symbols via the \( N \)-point IFFT as follows

\[
\tilde{s}_i(n) = \frac{1}{\sqrt{N}} \sum_{m \in \mathbb{Z}_N} s_i(m) e^{i2\pi mn/N}, \quad i \in \{1, 2, \ldots, N_T\} \text{ and } n \in \mathbb{Z}_N
\]  

where \( s_i(m) \) denotes the frequency-domain data symbol on the \( m \)-th subcarrier from the \( i \)-th transmit antenna, \( \tilde{s}_i(n) \) represents the time-domain data symbol on the \( n \)-th time-sample from the \( i \)-th transmit antenna, and \( \mathbb{Z}_K \triangleq \{0, 1, \ldots, K-1\} \). After the CP whose length should be enough sufficiently greater than maximum delay spread to avoid ISI is appended, the time-domain data symbols from each transmit antenna
undergo independently time-varying multipath fading channels as

\[ \tilde{r}_j(n) = \sum_{i=1}^{N_T} \sum_{l \in \mathbb{Z}_L} \tilde{h}_{ji}(n; l) \tilde{s}_i(n - l) + \tilde{z}_j(n), \quad j \in \{1, 2, \ldots, N_R\} \quad (3.2) \]

where \( \tilde{r}_j(n) \) and \( \tilde{z}_j(n) \) denotes the time-domain received symbols and the i.i.d. complex additive white Gaussian noise with zero-mean and variance of \( N_0/2 \) per dimension, respectively, on the \( j \)-th time-sample from the \( j \)-th receive antenna, and \( \tilde{h}_{ji}(n; l) \) signifies the \( l \)-th multipath channel linked from the \( i \)-th transmit antenna to the \( j \)-th receive antenna on the \( n \)-th time-sample under WSSUS assumption. After removing the CP, the time-domain received symbols are transformed into the frequency-domain received symbols using the \( N \)-point FFT as

\[ r_j(k) = \frac{1}{\sqrt{N}} \sum_{n \in \mathbb{Z}_N} \tilde{r}_j(n) e^{-i2\pi nk/N} \quad (3.3) \]

where \( r_j(k) \) denotes the frequency-domain received symbols on the \( k \)-th subcarrier from the \( j \)-th receive antenna, \( z_j(k) = \frac{1}{\sqrt{N}} \sum_{n \in \mathbb{Z}_N} \tilde{z}_j(n) e^{-i2\pi nk/N} \) means the frequency-domain AWGN on the \( k \)-th subcarrier from the \( j \)-th receive antenna, preserving its time-domain statistics by orthogonal transformation, and \( h_{j,i}(k, m) \) represents the \( m \)-th CFR on the \( k \)-th subcarrier from the link between \( i \)-th transmit antenna and \( j \)-th receive antenna, obtained by

\[ h_{j,i}(k, m) = \frac{1}{N} \sum_{n \in \mathbb{Z}_N} \sum_{l \in \mathbb{Z}_L} \tilde{h}_{ji}(n; l) e^{-i2\pi(l-n)(m+nk)/N}. \]

In the SC systems, the transmission and reception of MIMO can be expressed with matrix and vector notations, so Eq. (3.3) can be equiv-
alently rewritten as follows

\[
\begin{bmatrix}
R^1 \\
R^2 \\
\vdots \\
R^N
\end{bmatrix}
= \begin{bmatrix}
H^{1,1} & H^{1,2} & \cdots & H^{1,N} \\
H^{2,1} & H^{2,2} & \cdots & H^{2,N} \\
\vdots & \vdots & \ddots & \vdots \\
H^{N,1} & H^{N,2} & \cdots & H^{N,N}
\end{bmatrix}
\begin{bmatrix}
S^1 \\
S^2 \\
\vdots \\
S^N
\end{bmatrix}
+ \begin{bmatrix}
Z^1 \\
Z^2 \\
\vdots \\
Z^N
\end{bmatrix}
\tag{3.4}
\]

where \( R^k \triangleq [ r_1(k) \, \cdots \, r_{N_R}(k) ]^T, \) \( S^k \triangleq [ s_1(k) \, \cdots \, s_{N_T}(k) ]^T, \)
\( Z^k \triangleq [ z_1(k) \, \cdots \, z_{N_R}(k) ]^T, \) and

\[
H^{k,m} \triangleq \begin{bmatrix}
h_{1,1}(k,m) & h_{1,2}(k,m) & \cdots & h_{1,N_T}(k,m) \\
h_{2,1}(k,m) & h_{2,2}(k,m) & \cdots & h_{2,N_T}(k,m) \\
\vdots & \vdots & \ddots & \vdots \\
h_{N_R,1}(k,m) & h_{N_R,2}(k,m) & \cdots & h_{N_R,N_T}(k,m)
\end{bmatrix}.
\]

If the channel, on the other side, does not vary during OFDM symbol period, the ICI term in Eq. would be defunct as follows

\[
r_j(k) = \frac{1}{N} \sum_{i=1}^{N_T} \sum_{m \in \mathbb{Z}_N} \sum_{n \in \mathbb{Z}_N} \sum_{l \in \mathbb{Z}_L} \tilde{h}_{j,i}(l)e^{2\pi i (m(n-l)-nk)/N} s_i(m) + z_j(k)
= \frac{1}{N} \sum_{i=1}^{N_T} \sum_{m \in \mathbb{Z}_N} N \delta(m - k) \sum_{l \in \mathbb{Z}_L} \tilde{h}_{j,i}(l)e^{-i2\pi ml/N} s_i(m) + z_j(k)
= \sum_{i=1}^{N_T} \sum_{l \in \mathbb{Z}_L} \tilde{h}_{j,i}(l)e^{-i2\pi kl/N} s_i(k) + z_j(k)
= \sum_{i=1}^{N_T} \tilde{h}_{j,i}(k)s_i(k) + z_j(k).
\tag{3.5}\]

The above development implicates that all entries of \( H^{k,m} \) are zero for \( m \neq k \). Therefore, a procedure to evaluate the coded bit LLRs is comparable to that of the single-carrier systems since the input-output relation for each subcarrier would be simplified as

\[
R^k = H^{k,k} S^k + Z^k.
\tag{3.6}\]
Figure 3.3: Example of banded effective CFR matrix.

On the contrary, all entries of $H_{k,m}^{k,m}$ are no more zero for $m \neq k$ over the time-varying channel, whose energy distribution shows proportionally increasing as higher and higher normalized Doppler frequency $f_D T_s$.

3.2 Proposed IDD Receiver

In this section, the proposed IDD receiver is presented with the perfect CSI assumption at the receiver side. First, we provide the blockwise LMMSE detection considering considerably dominant ICIs. Then two different low complexity algorithms for LMMSE filter matrix inversion is introduced. In order to evaluate the extrinsic LLRs, finally, the soft-demapping is described, based on the blockwise LMMSE detection.

3.2.1 Blockwise LMMSE Detection

In this subsection, we detect each data symbol with the LMMSE approach in a blockwise fashion meaning that the block effective CFR matrix of size $N_R(2q + 1) \times N_T(4q + 1)$ is employed for each subcarrier instead of the full usage of it as depicted in Figure 3.3. In order to regard the input-output relation for each subcarrier in a blockwise point
of view, the blockwise system model can be yielded as

\[ \bar{R}^k \triangleq \bar{H}^k \bar{S}^k + \bar{W}^k + \bar{Z}^k, \tag{3.7} \]

where \( \bar{R}^k \triangleq [ (R^{(k-q)'}(k-q+1)') \cdots (R^{(k+q)'}T)]^T, \)
\( \bar{S}^k \triangleq [ (S^{(k-2q)'}(k-2q+1)') \cdots (S^{(k+2q)'}T)]^T, \)
\( \bar{W}^k \triangleq [ (W^{(k-q)'}(k-q+1)') \cdots (W^{(k+q)'}T)]^T, \)
\( \bar{Z}^k \triangleq [ (Z^{(k-q)'}(k-q+1)') \cdots (Z^{(k+q)'}T)]^T, \)
\( W^k \triangleq [ w_1(k) \ w_2(k) \cdots w_{N_R}(k) ]^T, \)
\( w_j(k) \triangleq \sum_{i=1}^{N_T} \sum_{m \in \mathcal{K}_k} h_{j,i}(k,m)s_i(m) \)
denotes the residual ICI term on the \( k \)-th subcarrier from the \( j \)-th receive antenna unconsidered in the blockwise LMMSE filter, \( \mathcal{K}_k \in \mathcal{Z}_N - \{ k-q, k-q+1, \ldots, k+q \} \), \( a' \triangleq a \mod N, \) and

\[
\bar{H}^k \triangleq \begin{bmatrix}
H^{(k-q)',(k-2q)'} & H^{(k-q)',(k-2q+1)'} & \cdots & H^{(k-q)',k'} \\
0_{N_R,NT} & H^{(k+1)',(k-2q+1)'} & \cdots & H^{(k+1)',(k+2q)'} \\
0_{N_R,NT} & \ddots & \cdots & 0_{N_R,NT} \\
H^{(k+1)',(k+1)'} & 0_{N_R,NT} & \cdots & 0_{N_R,NT} \\
\end{bmatrix}.
\]

Then the LMMSE detected symbol on the \( k \)-th subcarrier from the \( k \)-th subcarrier is obtained in the same way in Subsection 2.2.2. as follows

\[
s_i(k) = (\bar{h}^k_{2q,NT+i})^H (\Lambda^k + (1 - C_{s_i(k)}) \bar{h}^k_{2q,NT+i} (\bar{h}^k_{2q,NT+i})^H)^{-1} \\
\times (\bar{R}^k - \bar{H}^k E[\bar{S}^k] + E[s_i(k)] \bar{h}^k_{2q,NT+i}) \\
= F_i^k (\bar{h}^k_{2q,NT+i})^H (\Lambda^k)^{-1} (\bar{R}^k - \bar{H}^k E[\bar{S}^k] + E[s_i(k)] \bar{h}^k_{2q,NT+i}) \tag{3.8}
\]

where \( \Lambda^k \triangleq \bar{H}^k C_{S_{i(k)}} (\bar{H}^k)^H + R_{W_k} + R_{Z_k} \), \( R_{W_k} \) is given in Appendix 2, \( R_{Z_k} = N_0 I_{N_R(2q+1)} \), and \( F_i^k \triangleq \frac{1}{1+(1-C_{s_i(k)})((\bar{h}^k_{2q,NT+i})^H \Lambda^{-1} \bar{h}^k_{2q,NT+i}))} \). Also, \( E[s_i(k)] \)'s and \( C_{s_i(k)} \)'s can be calculated using Eqs. (2.10) and (2.11).
In Eq. (3.8), the main computational load is provoked by the matrix inverse operation \((\Lambda^k)^{-1}\). Meanwhile, the consecutively common submatrix can be observed on between the previous subcarrier and the present one (i.e., \(\tilde{\mathbf{H}}_{1:N_R,1:N_T+N_T(4q+1)} = \tilde{\mathbf{H}}_{1:N_R,1:N_T+N_T(4q+1)}\)). Using this feature, we propose two kinds of fast recursive algorithm in the next subsections.

### 3.2.2 Double-Pipelining Algorithm

The matrix inversion using submatrices, whose derivation is presented in Appendix A, is given by

\[
\begin{bmatrix}
    \mathbf{A} & \mathbf{B} \\
    \mathbf{B}^H & \mathbf{C}
\end{bmatrix}^{-1} =
\begin{bmatrix}
    \mathbf{X} & \mathbf{Y} \\
    \mathbf{Y}^H & \mathbf{Z}
\end{bmatrix} \quad (3.9)
\]

\[
\begin{bmatrix}
    \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\
    \tilde{\mathbf{B}}^H & \tilde{\mathbf{C}}
\end{bmatrix}^{-1} =
\begin{bmatrix}
    \tilde{\mathbf{X}} & \tilde{\mathbf{Y}} \\
    \tilde{\mathbf{Y}}^H & \tilde{\mathbf{Z}}
\end{bmatrix} \quad (3.10)
\]

where \(\mathbf{W} \triangleq \mathbf{B} \mathbf{V}, \mathbf{V} \triangleq \mathbf{C}^{-1}, \mathbf{X} \triangleq (\mathbf{A} - \mathbf{W} \mathbf{B}^H)^{-1}, \mathbf{Y} \triangleq -\mathbf{X} \mathbf{W}, \mathbf{Z} \triangleq \mathbf{V} - \mathbf{W}^H \mathbf{Y}, \tilde{\mathbf{W}} \triangleq \tilde{\mathbf{B}} \tilde{\mathbf{V}}, \tilde{\mathbf{V}} \triangleq \tilde{\mathbf{A}}^{-1}, \tilde{\mathbf{X}} \triangleq (\tilde{\mathbf{C}} - \tilde{\mathbf{W}} \mathbf{B})^{-1}, \tilde{\mathbf{Y}} \triangleq -\tilde{\mathbf{X}} \tilde{\mathbf{W}}, \) and \(\tilde{\mathbf{Z}} \triangleq \tilde{\mathbf{V}} - \tilde{\mathbf{W}}^H \tilde{\mathbf{Y}}\). In order to utilize Eqs. (3.9) and (3.10), the matrix inversion \((\Lambda^k)^{-1}\) is divided into two instances: \((\Lambda^{2k''})^{-1}\) and \((\Lambda^{2k''+1})^{-1}\) for \(k'' \in \mathbb{Z}_{N/2}\) since \(\Lambda^{2k''+1}_{N_R+1:N_R(2q+1),N_R+1:N_R(2q+1)}\) is equivalent to \(\Lambda^{2k''+1}_{1:2q:N_R,1:2q:N_R}\). For the even number subcarriers \(2k''\), the matrix inverse operation can be obtained as follows

\[
\begin{bmatrix}
    \mathbf{A} & \mathbf{B} \\
    \mathbf{B}^H & \mathbf{C}
\end{bmatrix}^{-1} =
\begin{bmatrix}
    \Lambda^{2k''}_{1:N_R,1:N_R} & \Lambda^{2k''}_{1:N_R,1:N_R(2q+1)} \\
    (\Lambda^{2k''}_{1:N_R,1:N_R(2q+1)})^H & \Lambda^{2k''}_{N_R+1:N_R(2q+1),1:N_R(2q+1)}
\end{bmatrix}^{-1}
\]

35
For the odd number subcarriers $2k'' + 1$, besides, the matrix inverse operation is represented by

$$
\begin{align*}
\left( \bar{\Lambda}_{2k'' + 1} \right)^{-1} & \triangleq \begin{bmatrix}
\bar{\Lambda} & \bar{B} \\
\bar{B}^H & \bar{C}
\end{bmatrix}^{-1} \\
& = \begin{bmatrix}
\bar{\Lambda}_{1:2q \cdot N_R},1:2q \cdot N_R & \bar{\Lambda}_{1:2q \cdot N_R},2q \cdot N_R + 1:2q \cdot N_R + 1;N_R(2q+1) \\
\bar{\Lambda}_{1:2q \cdot N_R},2q \cdot N_R + 1:2q \cdot N_R + 1;N_R(2q+1) \end{bmatrix}^{-1} \\
& = \begin{bmatrix}
\bar{X} \bar{Y} \\
\bar{Y}^H \bar{Z}
\end{bmatrix} \triangleq \bar{\Lambda}_{2k'' + 1}. \quad (3.12)
\end{align*}
$$

In Eqs. (3.9) and (3.10), $\mathbf{V}$ is identical with $\bar{\mathbf{V}}$, which is the novel motivation for the division into those two instances. Then the size of matrix inversion is reduced to $2q \cdot N_R \times 2q \cdot N_R$ from $N_R(2q+1) \times N_R(2q+1)$, and the matrix inverse operation with size of $2q \cdot N_R \times 2q \cdot N_R$ is worked only once every two subcarriers. Consequently, the process for the two consecutive subcarriers can be performed simultaneously in a pipelining fashion. The computational gain is accomplished for any $N_R$, $N_T$, and $q$, whose comparison will be shown in Subsection 3.3.3.
3.2.3 Fast Recursive Algorithm

The matrix inversion with its submatrices in Eqs. (3.9) and (3.10) is used again to contrive the fast recursive algorithm seeking the inverse matrix for the present subcarrier with that for the previous subcarrier. We define the inverse matrix for the previous subcarrier as follows

\[
\Lambda_{k-1}^{-1} = \begin{bmatrix} A & B \\ B^H & C \end{bmatrix}^{-1} = \begin{bmatrix} \Lambda_{1:N_R,1:N_R}^{-1} & \Lambda_{1:N_R,N_R+1:N_R(2q+1)}^{-1} \\ (\Lambda_{1:N_R,N_R+1:N_R(2q+1)}^{-1})^H & \Lambda_{N_R+1:N_R(2q+1),N_R+1:N_R(2q+1)}^{-1} \end{bmatrix}^{-1}
\]

\[
= \begin{bmatrix} \Lambda_{1:N_R,1:N_R}^{-1} & \Lambda_{1:N_R,N_R+1:N_R(2q+1)}^{-1} \\ (\Lambda_{1:N_R,N_R+1:N_R(2q+1)}^{-1})^H & \Lambda_{N_R+1:N_R(2q+1),N_R+1:N_R(2q+1)}^{-1} \end{bmatrix}
\]

\[
= \begin{bmatrix} X & Y \\ Y^H & Z \end{bmatrix} \triangleq \Lambda_{k-1}. \tag{3.13}
\]

Also, the matrix inversion for the present subcarrier is performed as

\[
\Lambda_k^{-1} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ \tilde{B}^H & \tilde{C} \end{bmatrix}^{-1} = \begin{bmatrix} \Lambda_{1:2qN_R,1:2qN_R} & \Lambda_{1:2qN_R,2qN_R+1:N_R(2q+1)} \\ (\Lambda_{1:2qN_R,2qN_R+1:N_R(2q+1)}^H)^H & \Lambda_{2qN_R+1:N_R(2q+1),2qN_R+1:N_R(2q+1)}^{-1} \end{bmatrix}^{-1}
\]

\[
= \begin{bmatrix} \Lambda_{1:2qN_R,1:2qN_R} & \Lambda_{1:2qN_R,2qN_R+1:N_R(2q+1)} \\ (\Lambda_{1:2qN_R,2qN_R+1:N_R(2q+1)}^H)^H & \Lambda_{2qN_R+1:N_R(2q+1),2qN_R+1:N_R(2q+1)} \end{bmatrix}
\]

\[
= \begin{bmatrix} \tilde{X} & \tilde{Y} \\ \tilde{Y}^H & \tilde{Z} \end{bmatrix} \triangleq \tilde{\Lambda}_k. \tag{3.14}
\]

The solution for \(\tilde{X}, \tilde{Y},\) and \(\tilde{Z}\) can be seen in Eq. (3.10), and, at the moment, \(\tilde{V}\) can be obtained from \((\Lambda_{N_R+1:N_R(2q+1),N_R+1:N_R(2q+1)}^{-1})^{-1}\). This
implies that it does not directly perform the matrix inverse operation of $\bar{A}$, but instead it can be obtained using the submatrix of the inverse matrix for the previous subcarrier $(\Lambda^{k-1})^{-1}$ as

$$\bar{V} = Z - ZB^H (A + BZB^H)^{-1} BZ$$

(3.15)

whose derivation is given in Appendix 1.2. Thus, the matrix inverse operation can be carried out in a recursive manner based on the inverse matrix for the previous subcarrier. Then the computational gain of this algorithm is achieved under $q \geq 2$, and it is better than that of the double-pipelining algorithm for $q \geq 3$. The details of these comparisons will be presented in Section 4.3.

3.2.4 Soft-Demapping

Based on the blockwise LMMSE detection, the extrinsic LLR is evaluated with the Gaussian approximation as

$$p(\hat{s}_i(k)|s_i(k) = \xi_a) \approx \psi_{\mu_{k,i,a},\sigma_{k,i,a}^2}(\hat{s}_i(k))$$

(3.16)

where $\mu_{k,i,a} \triangleq \mathbb{E}[\hat{s}_i(k)|s_i(k) = \xi_a]$ and $\sigma_{k,i,a}^2 \triangleq C_{\hat{s}_i(k)|s_i(k) = \xi_a}$. We can obtain these two conditional statistics as follows

$$\mu_{k,i,a} = F_i^k (\bar{H}_{2q-N_T+i}^k)^H (\Lambda^k)^{-1} (\mathbb{E}[\bar{R}^k|s_i(k) = \xi_a] - \bar{H}^k \mathbb{E}[\bar{S}^k])$$

$$+ \mathbb{E}[s_i(k)] \bar{H}_{2q-N_T+i}^k$$

$$= F_i^k \xi_a (\bar{H}_{2q-N_T+i}^k)^H (\Lambda^k)^{-1} \bar{H}_{2q-N_T+i}^k$$

(3.17)

$$\sigma_{k,i,a}^2 = (F_i^k)^2 (\bar{H}_{2q-N_T+i}^k)^H (\Lambda^k)^{-1} C_{\bar{R}^k|s_i(k) = \xi_a} (\Lambda^k)^{-1} \bar{H}_{2q-N_T+i}^k$$

(3.18)

where $C_{\bar{R}^k|s_i(k) = \xi_a} = \Lambda^k - C_{s_i(k)\bar{H}_{2q-N_T+i}^k}(\bar{H}_{2q-N_T+i}^k)^H$. Finally, the extrinsic LLR $L_E(c_{k,i,j})$ is computed with the max-log approximation to
reduce the number of addition as follows

\[ L_E(c_{k,i,j}) = \ln \frac{P(c_{k,i,j} = 1|\hat{s}_i(k))}{P(c_{k,i,j} = 0|\hat{s}_i(k))} - L_A(c_{k,i,j}) \]

\[ = \ln \sum_{\forall \xi_a: x_{a,j} = 1} p(\hat{s}_i(k)|s_i(k) = \xi_a) \prod_{\forall j' \neq j} P(c_{k,i,j'} = x_{a,j'}) \]

\[ \approx \max_{\forall \xi_a: x_{a,j} = 1} \pi_{k,i,j,a} - \max_{\forall \xi_a: x_{a,j} = 0} \pi_{k,i,j,a} \] (3.19)

where \( \pi_{k,i,j,a} \) is defined as

\[ \pi_{k,i,j,a} = -\frac{|\hat{s}_i(k) - \mu_{k,i,a}|^2}{\sigma_{k,i,a}^2} + \sum_{\forall j' \neq j} (x_{a,j'} - 0.5) \cdot L_A(c_{k,i,j'}). \]

3.3 Numerical Results and Discussion

In this section\(^1\), the performance of IDD for the different number of considered ICIs \( q \) at the normalized Doppler frequency of \( f_D T = 0.1, 0.3 \) is verified with the Monte-Carlo methodology in terms of average BER curves and EXIT charts in similar with Section 2. In the following, the computational complexity of the proposed algorithms for the matrix inverse operation \( (\Lambda^k)^{-1} \) is compared. The two-ray model is used for the multipath profile, which has uniform energy at \( 0T_s \) and \( 3T_s \). Each path generated using the Jakes’ model in [36] suffers independent Rayleigh fading channels under WSSUS assumption. The FFT size \( N \) is equal to 128 with CP length of \( N/8 \). The number of transmit and receive antennas is identically equal to four \( (N_T = N_R = 4) \). A non-recursive convolutional code of rate \( R_C = 1/2 \) with polynomials \( (133, 171) \) in octal form is used with QPSK. The spectral efficiency \( \eta \) is given by 4 bps/Hz, and 506 information bits are transmitted over a coding block with the interleaving size of 1024.

---

\(^1\)Here, we note OFDM parameters. \( N \) and \( N_{CP} \) denotes FFT size and CP length, respectively. In addition, \( f_D, T_s, \) and \( T \) are defined as Doppler frequency, sampling rate, and OFDM symbol period, respectively.
3.3.1 Error Performance

Figure 3.4 represents the performance comparison at $f_D T_s = 0.1$ between IDDs for $q \in \{0, 1, 2\}$ and MFB. The performance of IDDs as increasing the number of considered ICI is not distinguishably improved since the ICI power is not enough large compared to the noise power. In other words, the noise power dominantly affects the error performance due to its low operation SNR range. In higher-order modulation, however, it is surmised that the number of considered ICI is an important
Figure 3.5: BER Performance comparison of ICEDD vs. IDD for QPSK with imperfect CSI ($N_R = N_T = 4$, $R_C = 1/2$, $N = 128$, and $\eta = 4$ bps/Hz).

factor since the ICI power is comparable to the noise power in the relatively high operation SNR. In comparison with MFB, the SNR loss of IDDs at BER of $10^{-5}$ is given by about 0.7 dB.

Figure 3.5 shows the performance comparison over extremely high mobility as $f_D T_s = 0.3$ in difference with QPSK. In addition, we compares the performance depending on the consideration of the unconsidered ICI statistic. If the ICI statistic is considered, it is called unbiased in a estimation-theoretical manner. Otherwise, it is called biased. The
performance of unbiased IDD is slightly better than that of biased IDD. Its performance gain might be increased in higher-order modulation with the same reason mentioned in the previous subsection. The performance of IDD with \( q = 0 \) is very far from that of MFB as SNR loss of 3 dB. This is because that the ICI power is generated as much as the energy loss of the desired subcarrier, so the effective SINR is tremendously detracted. Meanwhile, the performance of IDD with \( q = 2 \) after four iterations enhanced as SNR of 1.5 dB compared with that of IDD with \( q = 0 \), and the SNR loss of it from MFB at BER of \( 10^{-4} \) is about 1.5 dB. Its convergence is inferior to that at \( f_D T_s = 0.1 \), so there needs more performance evaluations with more iterations in higher SNR to verify the convergence behavior approaching MFB.

### 3.3.2 EXIT Chart Analysis

In this subsection, the EXIT charts of the MIMO demapper and the SISO channel decoder are presented with respect to the error performance given in the previous subsection. The same methodology is used for this tool as presented in Chapter 2.

Figure 3.6 depicts the EXIT charts with \( f_D T_s = 0.1 \) at SNR of 8 dB to compare the convergence behavior of IDDs for \( q \in \{0, 1, 2\} \). There is almost no difference between them like as the error performance for them is quite similar.

In Figure 3.7, the EXIT charts with \( f_D T_s = 0.3 \) at SNR of 8 dB are presented under the three cases: biased IDD for \( q = 0 \), unbiased IDD for \( q = 0 \), and unbiased IDD for \( q = 2 \). Since the mutual information is closely related to the error performance in a proportional sense [31], we compare the convergence characteristic and the error performance in the following. The curve of unbiased IDD is upper than that of biased IDD with the same slope, which implies that the error performance of unbiased IDD is better as shown in the previous subsection. In addition,
the mutual information of IDD with $q = 2$ is considerably greater than that of IDD with $q = 0$. Consequently, the achievable error performance of IDD with the more the number of considered ICI would be less.

### 3.3.3 Complexity Analysis

In the proposed IDD, the main computational complexity is occurred by the matrix inverse operation $abc$ in Eq. (3.8), and we developed the low-complexity algorithms for that in Subsection 3.2.2 and 3.2.3. In this subsection, we compare the computational complexity between
them by counting real number operation. This evaluation, however, is fairly complicated due to several matrix operations, so we define some functions counting the real number operation with the same weight for real addition, real multiplication, and real division, according to each matrix operation in Table 3.1. Then the computational complexity of the direct matrix inverse operation $C_D$ is given by

$$C_D = \text{Inv}(N_R(2q + 1)).$$  \hspace{1cm} (3.20)
Table 3.1: Matrix Computational Complexity Evaluation Functions

<table>
<thead>
<tr>
<th>Matrix Operation</th>
<th>Function Name</th>
<th># of Real Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>Add($M, N$)</td>
<td>$2MN$</td>
</tr>
<tr>
<td>Multiplication</td>
<td>Mul($M, L, N$)</td>
<td>$8MLN - 2MN$</td>
</tr>
<tr>
<td>Inversion</td>
<td>Inv($M$)</td>
<td>$\frac{20}{3}M^3 + 9M^2 - \frac{14}{3}M$</td>
</tr>
</tbody>
</table>

That of the double-pipelining algorithm $C_{DP}$ is obtained as

$$C_{DP} = \frac{1}{2} \text{Inv}(2qN_R) + \text{Inv}(N_R) + \text{Mul}(2qN_R, 2qN_R, N_R)$$
$$+ \text{Mul}(2qN_R, N_R, 2qN_R) + \text{Mul}(N_R, 2qN_R, N_R)$$
$$+ \text{Mul}(N_R, N_R, 2qN_R) + \text{Add}(2qN_R, 2qN_R) + \text{Add}(N_R, N_R).$$

(3.21)

Finally, that of the fast-recursive algorithm $C_{FR}$ is evaluated as

$$C_{FR} = 2 \{ \text{Inv}(N_R) + \text{Mul}(2qN_R, 2qN_R, N_R)$$
$$+ \text{Mul}(2qN_R, N_R, 2qN_R) + \text{Mul}(N_R, 2qN_R, N_R)$$
$$+ \text{Mul}(N_R, N_R, 2qN_R) + \text{Add}(2qN_R, 2qN_R) + \text{Add}(N_R, N_R) \}$$

(3.22)

It is difficult to the evaluated results is explicitly understood, so we take some examples in terms of various simulation parameters by expressing the percentage of the real number of operation of those two proposed algorithms over that of the direct matrix inversion in Table 3.2. The two proposed algorithms shows the less computational complexity about as 60 % of the direct method. The computational complexity of the fast recursive algorithm is more rapidly decreasing as increasing $q$ compared to that of the double-pipelining one. Accordingly, the complexity of the fast recursive algorithm is less than that of the double-pipelining one for $q \geq 3$. 
Table 3.2: Examples of Complexity for the Proposed Algorithms

<table>
<thead>
<tr>
<th>$N_T$</th>
<th>$N_R$</th>
<th>$q$</th>
<th>$\frac{C_{DP}}{C_D} \times 100%$</th>
<th>$\frac{C_{FR}}{C_D} \times 100%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>60.5</td>
<td>68.5</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>3</td>
<td>58.7</td>
<td>53.6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>57.4</td>
<td>44.0</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>62.5</td>
<td>73.1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>3</td>
<td>59.9</td>
<td>56.3</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>58.1</td>
<td>45.7</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>2</td>
<td>62.5</td>
<td>73.1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>3</td>
<td>59.9</td>
<td>56.2</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>58.1</td>
<td>45.7</td>
</tr>
</tbody>
</table>
4. Conclusion and Future Work

In this thesis, we have developed the iterative channel estimation, detection, and decoding for coded MIMO systems on SC- and OFDM-transmission in a linear fashion.

In practical coded MIMO systems, the pilot-aided channel estimation error severely degrades error performance due to their low operation SNR range. In this respect, we propose the ICEDD for coded MIMO systems on SC transmission over block fading channels in a manner of statistical theory. The LMMSE channel estimator is performed with $N_T$ pilot symbols at the initial step, and it utilizes the soft information of data symbols fed back from the SISO channel decoder as well as the pilot symbols for each iteration. The LMMSE detector is derived with the consideration of the channel estimation error statistic, and the number of matrix inversion which is a main computational burden factor is reduced to one for detecting $N_T$ data symbols. In addition, the computational complexity to obtain the LMMSE filter is further reduced linearizing the matrix inverse operation with the help of the Sherman-Morrison formula. Finally, the soft-demapper under the LMMSE detector is presented with the max-log approximation, which yields the extrinsic LLR fed into the channel decoder. The simulation results demonstrate that the proposed algorithm outperforms the IDD without channel estimation refinement and their performance nearly approaches to that of MFB. The SNR loss far from the MFB for QPSK at BER of $10^{-6}$ is given about 0.4 dB, and that for 16-QAM at BER of $10^{-8}$ is less than 0.3 dB. It is emphasized that our proposed algorithm would be suitable for the next generation wireless communication systems in practical point of view. For the further researches on this
problem, we will elaborately observe the convergence characteristic of ICEDD to prevent vain computational effort by adaptively determining the remaining iteration progression.

For coded MIMO-OFDM systems, we consider the extremely high-mobility environment such as $f_D T_s = 0.1, 0.3$. Over the time-varying channels, the energy of desired subcarrier is deteriorated, and its loss is dispersed into nearly adjacent subcarriers, called ICI. In order to resolve this problem, the IDD is proposed using the blockwise LMMSE detection which takes several dominantly effective ICIs to avert the full matrix inversion of size $N$. In addition, the two low-complexity matrix inverse algorithms is developed using the fact that the consecutively common matrix exists on the present and next subcarrier. They reduces the computational complexity about as 40% of the direct matrix inversion. The simulation result at $f_D T_s = 0.1$ shows that the SNR loss of IDD from the MFB is about 0.7 dB at BER of $10^{-5}$. Over quite time-varying channels, $f_D T_s = 0.3$, the number of considered ICI is more required to close the MFB. The SNR loss of IDD with $q = 2$ from the MFB is given by 1.5 dB at BER of $10^{-4}$. To verify the performance loss, we needs to increase the number of iteration and to observe the high SNR regime. In addition, for future work, these approaches will be practically considered with the channel estimation, and the error performance analysis over mobility might be regarded as a good challenge for future wireless mobile communications.
Appendix A. Matrix Inversion

1.1 Matrix Inversion with its Submatrices

Let us assume that a Hermitian matrix consists of four submatrices as follows

\[
M \triangleq \begin{bmatrix} A & B \\ B^H & C \end{bmatrix}.
\]  

(A.1)

Also, its inverse matrix can be constituted with four submatrices as

\[
M^{-1} \triangleq \begin{bmatrix} X & Y \\ Y^H & Z \end{bmatrix}.
\]  

(A.2)

Then the multiplication of the matrix and its inverse matrix becomes naturally the identity matrix as

\[
MM^{-1} = \begin{bmatrix} A & B \\ B^H & C \end{bmatrix}
\begin{bmatrix} X & Y \\ Y^H & Z \end{bmatrix} = \begin{bmatrix} AX + BY^H = I \\ B^HX + CY^H = 0 \end{bmatrix}.
\]  

(A.3)

From Eq. (A.3), \( Y^H \) and \( Y \) are given by

\[
Y^H = -C^{-1}B^HX, \quad Y = -A^{-1}BZ.
\]  

(A.4)

(A.5)

They are substituted in \( AX + BY^H = I \) and \( B^HY + CZ = I \), respectively, as follows

\[
X = (A - BC^{-1}B^H)^{-1}
\]  

(A.6)

\[
= A^{-1} + A^{-1}B(C - B^HA^{-1}B)^{-1}B^HA^{-1},
\]  

(A.7)

\[
Z = (C - B^HA^{-1}B)^{-1}
\]  

(A.8)

\[
= C^{-1} + C^{-1}B^H(A - BC^{-1}B^H)^{-1}BC^{-1}.
\]  

(A.9)
Using Eqs. (A.4) to (A.9), the inverse matrix of \( M \) using its submatrices can be obtained as

\[
M^{-1} = \begin{bmatrix}
X & Y \\
Y^H & Z
\end{bmatrix}
\]

(A.10)

\[
= \begin{bmatrix}
\bar{X} & \bar{Y} \\
\bar{Y}^H & \bar{Z}
\end{bmatrix}
\]

(A.11)

where \( W \triangleq BV, \ V \triangleq C^{-1}, \ X \triangleq (A - WB^H)^{-1}, \ Y \triangleq -XW, \ Z \triangleq V - W^HY, \ \bar{W} \triangleq B^H\bar{V}, \ \bar{V} \triangleq A^{-1}, \ \bar{X} \triangleq (C - \bar{W}B)^{-1}, \ \bar{Y} \triangleq -\bar{X}W, \) and \( \bar{Z} \triangleq \bar{V} - \bar{W}^H\bar{Y} \).

1.2 Submatrix Inversion from its Inverse Matrix

In this section, the inverse matrix of \( C^{-1} \), which is the submatrix of \( M \) in Eq. (A.10), is derived from its inverse matrix \( M^{-1} \). We start from \( Z = C^{-1} - W^HY \) in Eq as follows

\[
C^{-1} = Z + W^HY \\
= Z - C^{-1}B^H(A - BC^{-1}B^H)^{-1}BC^{-1} \\
= Z - C^{-1}B^HA^{-1}B(C - B^HA^{-1}B)^{-1}.
\]

(A.12)

After some mathematical manipulations, which are not noted here due to their irksomeness, Eq. (A.12) can be simplified as

\[
I = ZC - ZB^HA^{-1}B.
\]

(A.13)

Finally, the above equation is rearranged in terms of the submatrix of \( M^{-1} \) and the submatrices of \( M \) to obtain \( C^{-1} \) as

\[
C^{-1} = (I + ZB^HA^{-1}B)^{-1}Z \\
= (Z^{-1} + B^HA^{-1}B)^{-1} \\
= Z - ZB^H(A + BZB^H)^{-1}BZ.
\]

(A.14)
Appendix B. Statistics of ICI

In this chapter, we derive the statistics of the unconsidered ICI, \( \Omega \equiv R_{W_k} \), under the WSSUS assumption: energy term on the diagonal of \( \Omega \) and colored term on the off-diagonal of \( \Omega \).

2.1 Energy Term of ICI

The energy of the unconsidered ICI can be evaluated as

\[
\omega_{uu} \equiv \sum_{i=1}^{N_T} \sum_{q=1}^q (E_s - R_{h_{ji}(k,m)s_i(m)}(\theta)), \quad u \in \{1, 2, \ldots, N_R(2q + 1)\}
\]  

(B.1)

where \( \theta \equiv m - k \) and \( R_{h_{ji}(k,m)s_i(m)}(\theta) \) is obtained as follows

\[
R_{h_{ji}(k,m)s_i(m)}(\theta) = E[h_{ji}(k, m)s_i(m)h_{ji}^*(k, m)]
= E[s_i(m)s_i^*(m)]E[h_{ji}(k, m)h_{ji}^*(k, m)]
= \frac{E_s}{N^2} \sum_{n \in Z_N} \sum_{l \in Z_L} \sum_{n' \in Z_N} \sum_{l' \in Z_L} E[h_{ji}(n; l)h_{ji}^*(n'; l')]
\times e^{-2\pi i(n-n'+m)/N} e^{-2\pi i(n-n')k/N}
\]

(B.2)

\[
= \frac{E_s}{N^2} \sum_{n \in Z_N} \sum_{n' \in Z_N} J_0(2\pi f_D T_s(n' - n)) \sigma_r^2 \delta(l - l')
\times e^{-2\pi i(n-n'+m)/N} e^{-2\pi i(n-n')k/N}
\]

\[
= \frac{E_s}{N^2} \sum_{n \in Z_N} \sum_{n' \in Z_N} J_0(2\pi f_D T_s(n' - n)) e^{-2\pi i(n-n')(m-k)/N}
\]

\[
= \frac{E_s}{N^2} \sum_{n \in Z_N} \sum_{n' \in Z_N} J_0(2\pi f_D T_s(n' - n)) e^{-2\pi i(n-n')\theta/N}.
\]
In the above expressions, $\sigma^2_l$ means the $l$-th multipath power profile under $\sum_{l \in \mathbb{Z}_L} \sigma^2_l = 1$. Substituting $R_{h_{ji}(k,m)s_i(m)}(\theta)$ in Eq. (B.1) finalizes as

$$\omega_{uu} = \sum_{i=1}^{N_T} \sum_{\theta=-q}^{q} (E_s - R_{h_{ji}(k,m)s_i(m)}(\theta))$$

$$= N_T \left( E_s - \frac{E_s}{N^2} \sum_{n \in \mathbb{Z}_N} \sum_{n' \in \mathbb{Z}_N} J_0(2\pi f_D T_s(n' - n)) e^{-2\pi(n'-n)\theta/N} \right)$$

$$= N_T E_s \left\{ 1 - \frac{1}{N^2} \left( N(2q + 1) + 2 \sum_{n=1}^{N-1} (N - n) \phi_n \cdot J_0(2\pi f_D T_s n) \right) \right\}$$

(B.3)

where $\phi_n = \frac{\cos(2\pi nq/N) - \cos(2\pi n(q+1)/N)}{1 - \cos(2\pi n/N)}$.

### 2.2 Colored Term of ICI

In this section, the colored term of the unconsidered ICI is derived as follows

$$\omega_{uu'} \triangleq \begin{cases} 
\sum_{i=1}^{N_T} \left( \frac{N}{2} - 1 \right) \sum_{\theta=-N/2}^{N/2} R_{[h_{ji}(k,m)s_i(m)]h_{ji}(k,v,m)s_i(m)}(\theta) \\
- \sum_{\theta=-q-v}^{q} R_{[h_{ji}(k,m)s_i(m)]h_{ji}(k-v,m)s_i(m)}(\theta) \\
\omega_{uu'+N_R \cdot v}^* \text{ if } u = u_v \text{ and } u' = u_v + N_R \cdot v, \\
0 \text{ otherwise}
\end{cases}$$

(B.4)

where $u \in \{1, 2, \ldots, N_R(2q + 1)\}$, $u' \in \{1, 2, \ldots, N_R(2q + 1)\}$, $u' \neq u$, $v \in \{1, 2, \ldots, 2q\}$, $u_v \in \{1, 2, \ldots, N_R(2q + 1 - v)\}$, and $\theta \triangleq m - k$. 

52
\[ R[h_{ji}(k, m)s_i(m)][h_{ji}(k-v, m)s_i(m)](\theta) \] in Eq. (B.4) is given by

\[
R[h_{ji}(k, m)s_i(m)][h_{ji}(k-v, m)s_i(m)](\theta) \\
= E[h_{ji}(k, m)s_i(m)h_{ji}^*(k-v, m)] \\
= E[s_i(m)s_i^*(m)]E[h_{ji}(k, m)h_{ji}^*(k-v, m)] \\
= \frac{E_s}{N^2} \sum_{n \in \mathbb{Z}_N} \sum_{l \in \mathbb{Z}_L} \sum_{n' \in \mathbb{Z}_N} \sum_{l' \in \mathbb{Z}_L} E[h_{ji}(n; l)h_{ji}^*(n'; l')] \\
\times e^{-\frac{2\pi}{L}(l-n-l'+n')/N} e^{-\frac{2\pi}{N}(nk-n'k)/N} \\
= \frac{E_s}{N^2} \sum_{n \in \mathbb{Z}_N} \sum_{n' \in \mathbb{Z}_N} J_0(2\pi f_D T_s(n' - n))e^{-\frac{2\pi}{N}(n'-n)(m-k)/N} e^{-\frac{2\pi}{N}(n'-n)\theta + N'v} \\
= \frac{E_s}{N^2} \sum_{n \in \mathbb{Z}_N} \sum_{n' \in \mathbb{Z}_N} J_0(2\pi f_D T_s(n' - n))e^{-\frac{2\pi}{N}(n'-n)\theta + N'v}/N \quad (B.5)
\]

Then we can attain the final expression by substituting Eq. (B.5) in Eq. (B.4) as

\[
\omega_{uu'} = \begin{cases} 
-\frac{N_f E_s}{N^2} \sum_{n \in \mathbb{Z}_N} \sum_{n' \in \mathbb{Z}_N} J_0(2\pi f_D T_s(n' - n)) \\
\times e^{-\frac{2\pi}{N}(n'-n)\theta + N'v}/N \quad \text{if } u = (u_v + N_R \cdot v) \text{ and } u' = u_v, \\
\omega_{(u_v+N_R \cdot v)u_v}^* \quad \text{if } u = u_v \text{ and } u' = (u_v + N_R \cdot v), \\
0 \quad \text{otherwise.} 
\end{cases} \quad (B.6)
\]
단일반송과 및 직교주파수분할다중 기반의 부호화된 다중안테나 시스템을 위한 반복 수신기 설계

신중립

다중안테나(MIMO) 기술은 주파수재영역과 전력의 증가 없이 전송률 향상과 함께 높은 신뢰도를 성취할 수 있어 활발한 연구가 진행되고 있다. 대표적인 V-BLAST (Vertical Bell Labs Layered Space-Time) 전송 구조는 간단하면서 송신안테나 개수만큼의 공간다중 이득과 수신안테나 개수만큼의 다이버스이득을 얻을 수 있기 때문에 IEEE 802.16e나 3GPP-LTE 표준에도 적용되어 차세대 이동통신 기술로서 각광을 받고 있다. 다중안테나 기법은 채널부호화기법과 결합하여 상당한 시스템 성능을 제공한다. 특히, space-time bit-interleaved coded modulation (ST-BICM) 시스템은 코드율, 변조레벨, 송신안테나 개수에 따라 설계함으로써 전송률의 호환이 가변성이 뛰어나고 송신구조가 간단하다. 하지만 수신구조의 경우 각 송신안테나로부터 들어오는 간섭 신호의 영향을 처리하기 때문에 송신구조에 비해 상당한 복잡성을 갖는다. 이를 위한 많은 수신구조가 연구 중이고, matched filter bound (MFB)에 근접하는 최적의 성능을 보이는 iterative detection and decoding (IDD) 수신기에 대한 연구가 주목을 받고 있다. IDD 수신기법은 크게 두 가지로 나뉜다. 우선, maximum a posteriori probability (MAP)를 위한 sphere detection (SD) 기법이 대표적이다. SD 기법은 tree 검색 기반의 알고리즘으로 최적의 성능을 가져오지만, 복잡도가 상당히 높기 때문에 구현측면에서 비효율적이다. 한편, linear minimum mean squared error (LMMSE) 기법은 채널복호기로부터 받는 사전정보를 바탕으로 선형적인 연산을 통하여 낮은 복잡도를 가지면서 몇 회의 반복과정을 통하여 최적의 성능을 보인다.
기존의 ST-BICM 시스템을 위한 IDD 수신기는 수신기에서 채널 정보가 완벽하다는 가정과 함께 신호검출기법이 제안되었다. 하지만 실제 시스템에서는 파일럿 심볼로부터 채널추정을 수행하기 때문에, 낮은 작동 signal-to-noise ratio (SNR) 범위를 갖는 IDD 기반의 ST-BICM 시스템에서의 채널추정 오차는 상당히 크고, 이로 인하여 심각한 성능 열화가 초래된다. 본 연구에서는 채널추정 오차를 완화시키기 위하여 IDD 수신기에 반복 LMMSE 채널추정 과정을 포함시킨다. 채널복호기로부터 받은 신호정보를 바탕으로 LMMSE 채널추정을 다시 수행하고, 이를 바탕으로 LMMSE 신호검출을 수행하여 상당한 성능 향상을 얻을 수 있다. LMMSE 신호검출 시 채널추정의 Bayesian MSE를 고려하면 좀 더 SNR 이득을 얻을 수 있고, 높은 변조레벨에서의 error-floor를 제거할 수 있다. 이와 더불어, 실제 환경에서의 저 복잡도 구현을 위하여 효율적인 Sherman-Morrison 공식을 적용한 신호검출기와 demapper를 제안한다.

최근 연구되는 대부분의 다중안테나 시스템은 광대역 통신을 위하여 직교주파분할다중(OFDM) 기법과 결합되어 사용된다. OFDM 기법은 다중경로채널에 의한 주파수선택적 채널에 강하고, 부호화된 다중안테나 기법과 함께 사용될 경우 시간, 공간, 주파수 3차원에 대한 다이버시티 이득을 얻어 단일송수신 시스템에 비해 향상된 성능을 가져온다. 하지만 시간에 따른 채널변화가 심할 경우 직교성이 깨지고 인접부분송수신간섭(ICI)로 인한 성능범화가 발생한다. 인접송신 안테나간섭(ITAI)과 ICI를 효과적으로 제거하고, 높은 신뢰도의 신호검출을 위해서는 IDD 수신기의 작용이 필수 불가결하다. 하지만 fast Fourier transform (FFT) 사이즈에 비례하는 유 효채널행렬에 의해 상당한 복잡도를 요구하고, 실시간 서비스를 제공하기 위한 실제 시스템에서는 적용될 수 없다. 영향력 있는 ICI는 바로 인접한 일부 부반송파에만 해당되기 때문에, 영향력 있는 ICI만 고려한 부분 유 효채널행렬에 대해 IDD를 적용하면 저 복잡도를 보이면서 높은 성능을 가질 수 있다. 특히, 신호검출에 필요한 LMMSE 필터 연산을 제귀적으로 수행함으로써 실제 시스템에 적용 가능한 낮은 복잡도를 보여준다.
References


57


